Margita Vajsáblová

Mongeova projekcia

- polohové úlohy

Základné pojmy a obraz bodu v Mongeovej projekcii

Priemetne:

π – pôdorysňa, ${}^1s \perp \pi$,

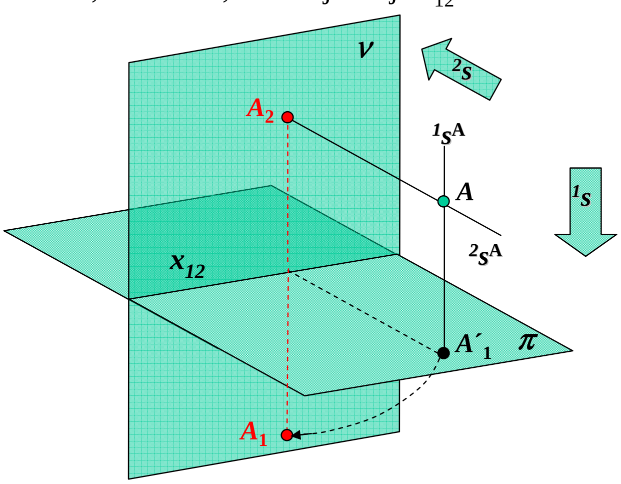
ν – nárysňa, ${}^2s \perp \nu$,

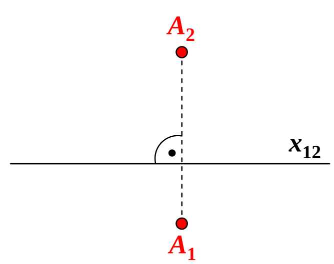
$\pi \perp \nu$, $\pi \cap \nu = x$, označujeme ju x_{12} – základnica.

***Priemety bodu** A:*

$\pi \cap {}^1s^A = A'_1$ – pôdorys bodu A, ${}^1s^A$: $A \in {}^1s^A$, ${}^1s^A \perp \pi$,

$\nu \cap {}^2s^A = A_2$ – nárys bodu A, ${}^2s^A$: $A \in {}^2s^A$, ${}^2s^A \perp \nu$.





Združenie priemetní:

π otočíme do ν okolo x, A'_1 sa otočí do A_1,

A_1, A_2 – *združené priemety bodu* A,

platí $A_1A_2 \perp x_{12}$,

A_1A_2 – ordinála bodu A.

Definícia: Bijektívne zobrazenie, ktoré každému bodu $A \in E_3$ priradí združené priemety $[A_1, A_2]$, $A_1A_2 \perp x_{12}$, voláme ***kolmé premietanie na dve navzájom kolmé priemetne – Mongeova projekcia.***

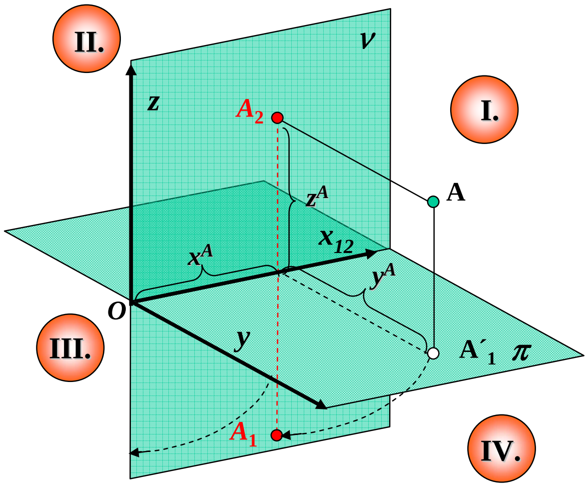
Obraz bodu v Mongeovej projekcii

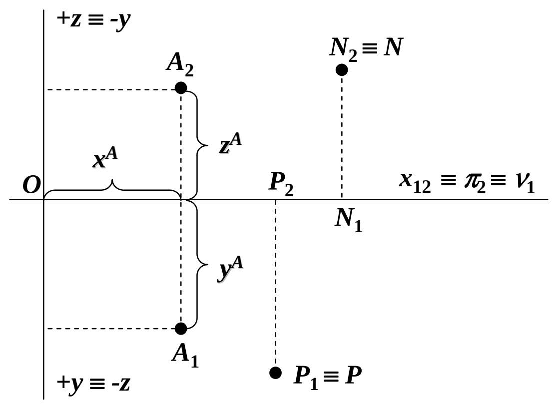
Pravouhlá súradnicová sústava:

$x, y \subset \pi$, $A_1\,[x^A, y^A]$, kde x je základnica,

$x, z \subset \nu$, $A_2\,[x^A, z^A]$,

V združení priemetní: $+z \equiv -y$, $+y \equiv -z$





Kvadranty: π a ν rozdeľujú E_3 na 4 kvadranty
I. kvadrant $y > 0$, $z > 0$, II. kvadrant $y < 0$, $z > 0$,
III. kvadrant $y < 0$, $z < 0$, IV. kvadrant $y > 0$, $z < 0$.

Body priemetní:

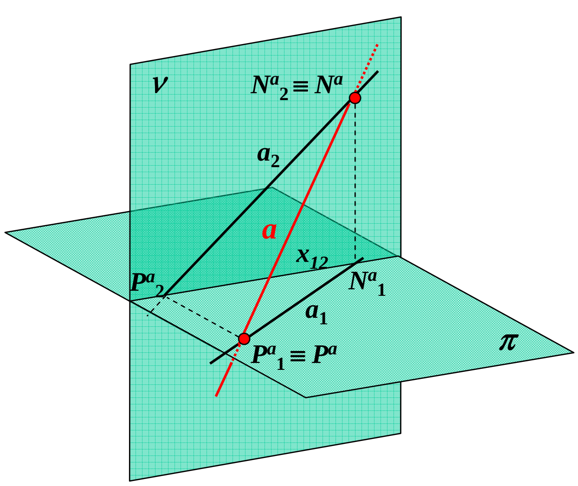
$P \in \pi \Rightarrow P_1 \equiv P$, $P_2 \in x_{12}$, $z^P = 0$

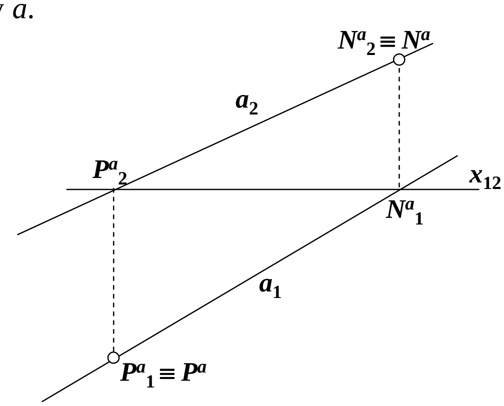
$N \in \nu \Rightarrow N_1 \in x_{12}$, $N_2 \equiv N$, $y^N = 0$

Obraz priamky v Mongeovej projekcii

Stopníky priamky: $a \cap \pi = P^a$ – pôdorysný stopník priamky a,

$a \cap \nu = N^a$ – nárysný stopník priamky a.





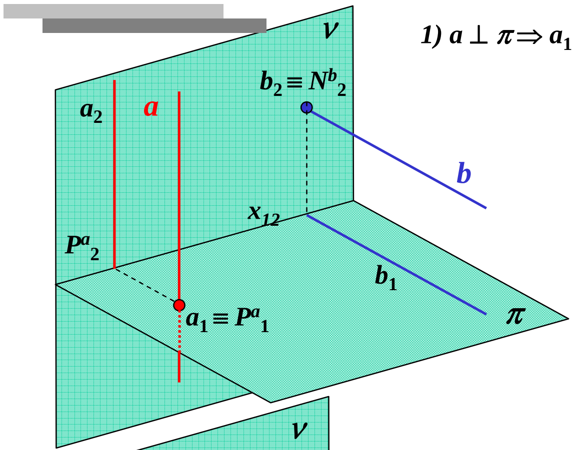
Konštrukcia pôdorysného stopníka:

$a_2 \cap x_{12} = P^a_2$, $P_1 \in a_1$

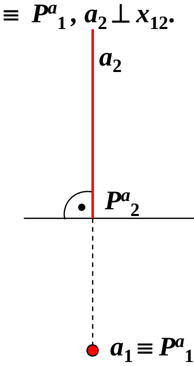
Konštrukcia nárysného stopníka:

$a_1 \cap x_{12} = N^a_1$, $N_2 \in a_2$.

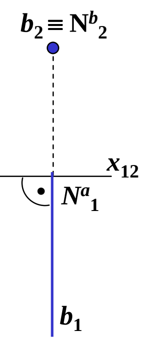
Obraz priamok v Mongeovej projekcii

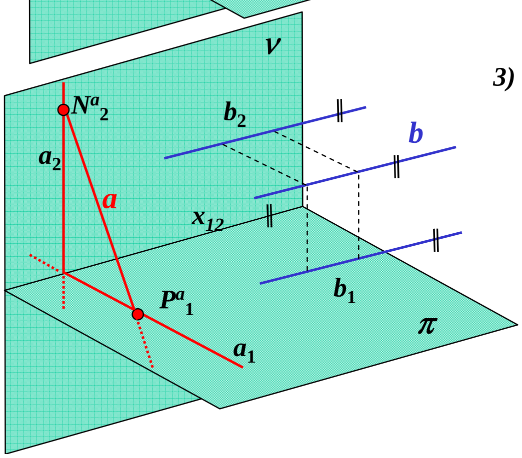


1) $a \perp \pi \Rightarrow a_1 \equiv P^a_1,\ a_2 \perp x_{12}$.

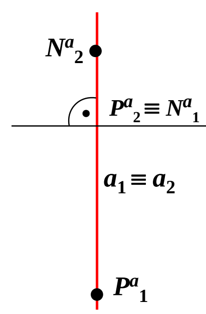


2) $b \perp \nu \Rightarrow b_2 \equiv N^b_2,\ b_1 \perp x_{1}$.

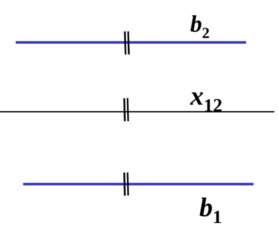




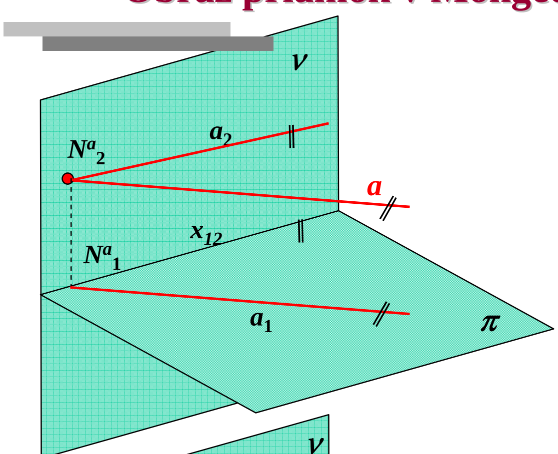
3) $a \perp x \Rightarrow a_1 \equiv a_2 \perp x_{12}$.



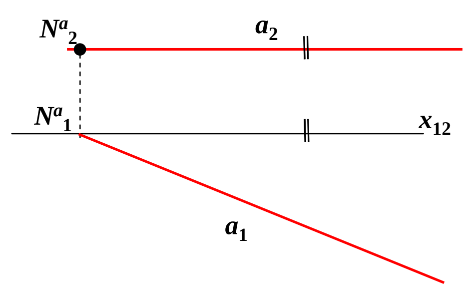
4) $b \parallel x \Rightarrow b_1 \parallel b_2 \parallel x_{12}$.

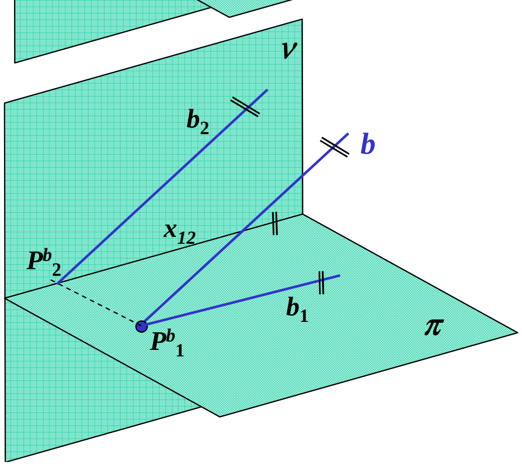


Obraz priamok v Mongeovej projekcii

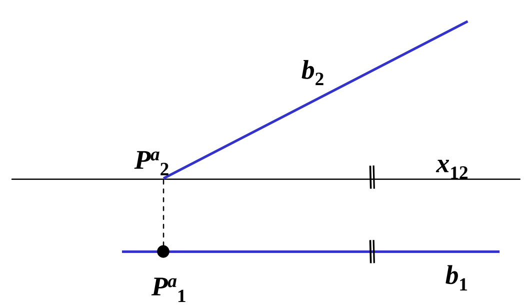


5) $a \parallel \pi \Rightarrow a_2 \parallel x_{12}$





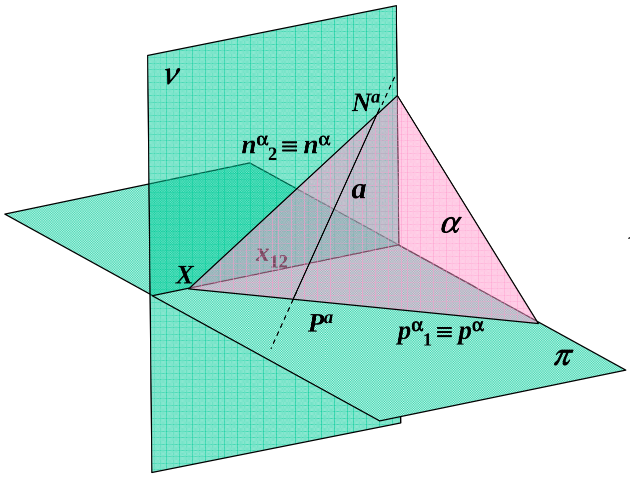
6) $b \parallel \nu \Rightarrow b_1 \parallel x_{12}$

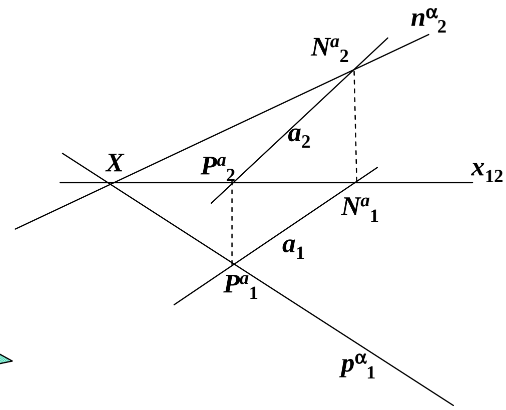


Obraz roviny v Mongeovej projekcii

Stopy roviny: $\alpha \cap \pi = p^{\alpha}$ – pôdorysná stopa roviny α,

$\alpha \cap \nu = n^{\alpha}$ – nárysná stopa roviny α. Ak existuje $X = p^{\alpha} \cap n^{\alpha}$, potom $X \in x$.

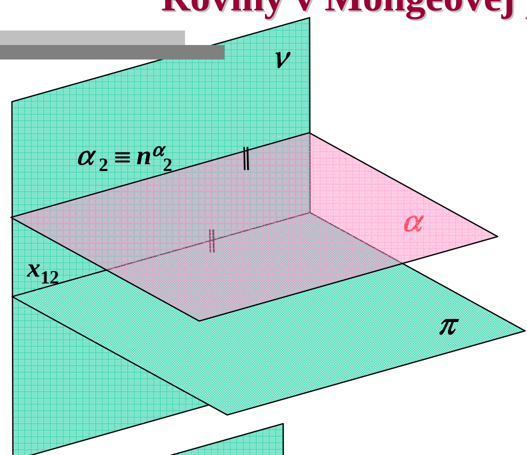




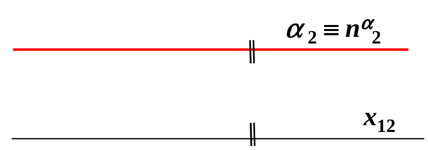
Ak priamka leží v rovine a má stopníky, potom jej pôdorysný stopník leží na pôdorysnej a nárysný na nárysnej stope roviny:

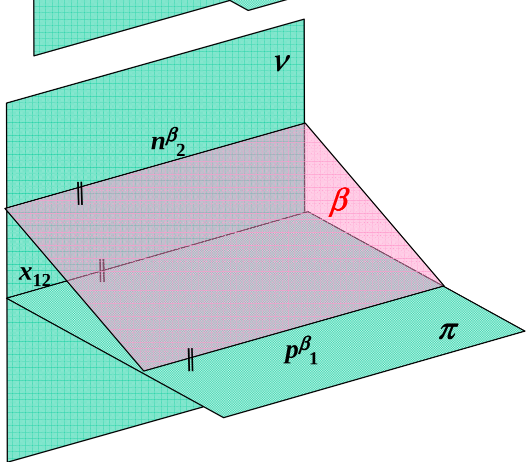
$$P^{a}_{1} \in p^{\alpha}_{1}\,,\; N^{a}_{2} \in n^{\alpha}_{2}$$

Roviny v Mongeovej projekcii

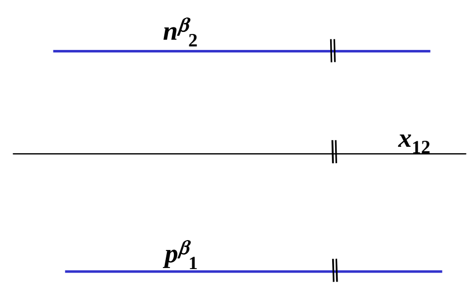


1) $\alpha \parallel \pi \Rightarrow \alpha_2 \equiv n^{\alpha}_2 \parallel x_{12}$

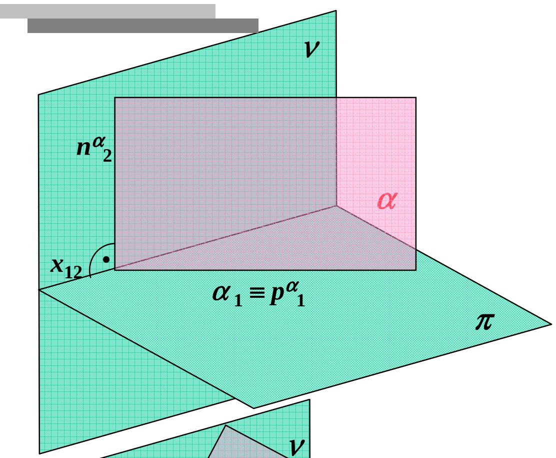




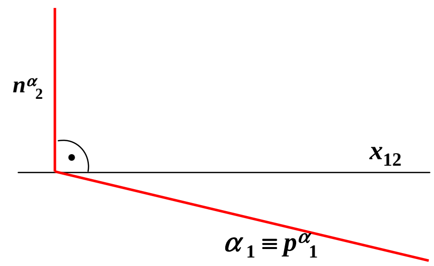
2) $\beta \parallel x \Rightarrow p^{\beta}_1 \parallel n^{\beta}_2 \parallel x_{12}$

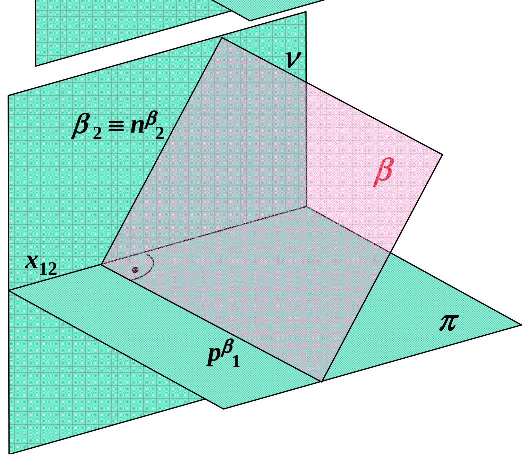


Roviny v Mongeovej projekcii

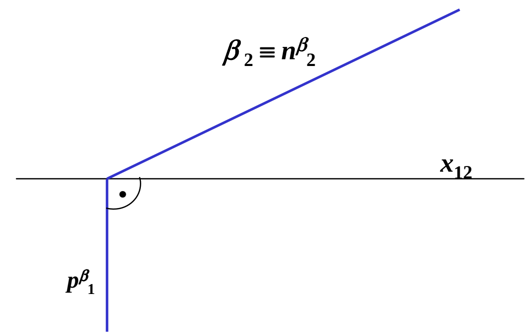


3) $\alpha \perp \pi \Rightarrow \alpha_1 \equiv p^{\alpha}_1,\ n^{\alpha}_2 \perp x_{12}$





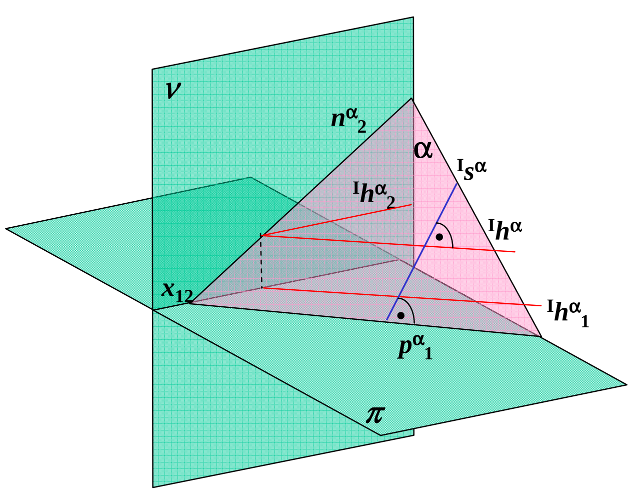
4) $\beta \perp \nu \Rightarrow \beta_2 \equiv n^{\beta}_2,\ p^{\beta}_1 \perp x_{12}$

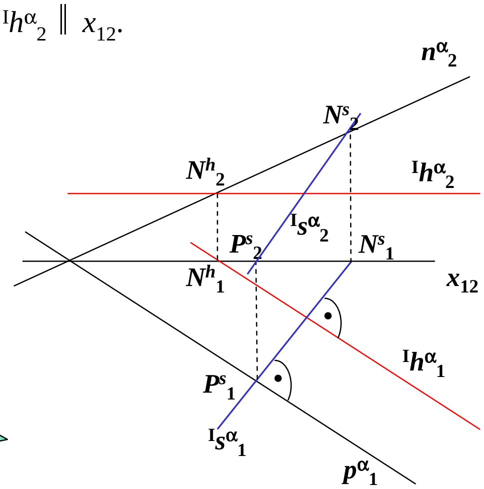


Hlavné a spádové priamky roviny v Mongeovej projekcii

Hlavné priamky I. osnovy roviny α: ${}^{I}h^{\alpha} \parallel \pi$,

${}^{I}h^{\alpha}_1 \parallel p^{\alpha}_1,\ {}^{I}h^{\alpha}_2 \parallel x_{12}$.





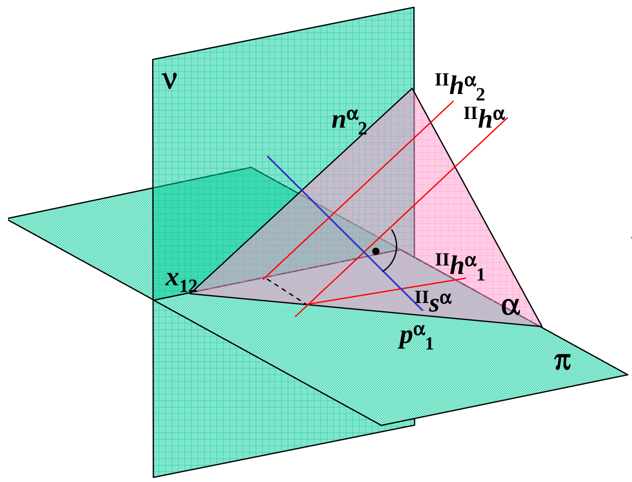
Spádové priamky I. osnovy roviny α: ${}^{I}s^{\alpha} \perp {}^{I}h^{\alpha}\ (p^{\alpha})$,

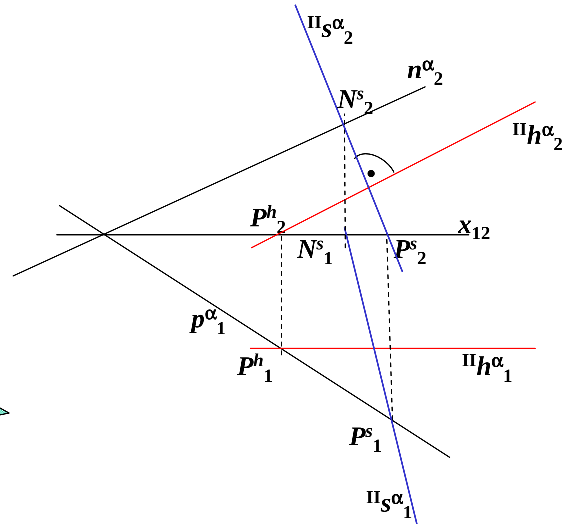
${}^{I}s^{\alpha}_1 \perp p^{\alpha}_1,\ {}^{I}s^{\alpha}_2 = P^{s}_2 N^{s}_2$.

Hlavné a spádové priamky roviny v Mongeovej projekcii

Hlavné priamky II. osnovy roviny α: $^{II}h^{\alpha} \parallel \nu$

$^{II}h^{\alpha}_1 \parallel x_{12}$, $^{II}h^{\alpha}_2 \parallel n^{\alpha}_2$



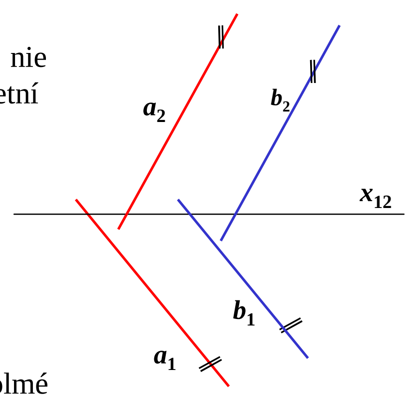


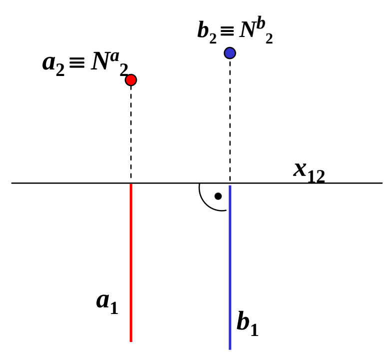
Spádové priamky II. osnovy roviny α: $^{II}s^{\alpha} \perp {}^{II}h^{\alpha}\,(n^{\alpha})$

$^{II}s^{\alpha}_2 \perp n^{\alpha}_2$, $^{II}s^{\alpha}_1 = P^s_1 N^s_1$

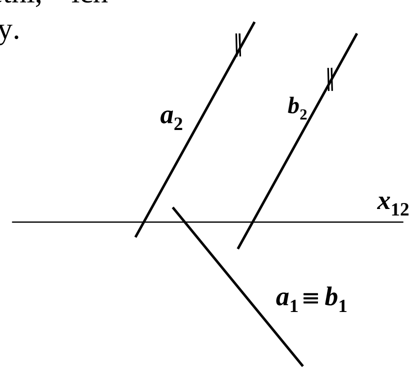
Vzájomná poloha 2 priamok v Mongeovej projekcii

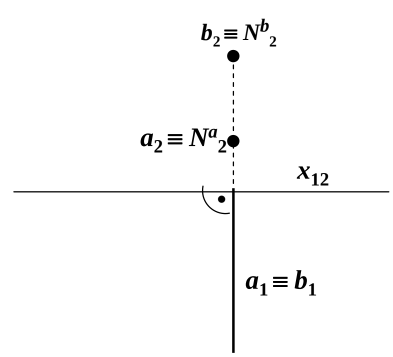
1) Rovnobežné priamky a, b, ak nie sú kolmé na žiadnu z priemetní a $a \parallel b \Rightarrow a_1 \parallel b_1, a_2 \parallel b_2$.





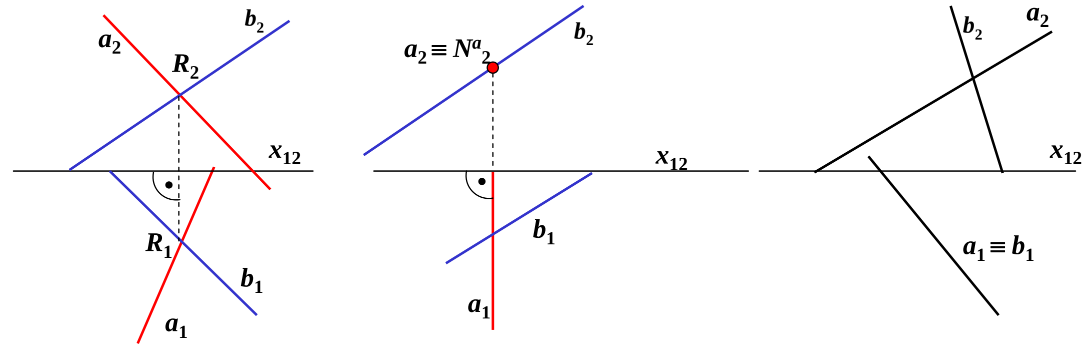
Ak sú rovnobežné priamky kolmé na niektorú z priemetní, ich priemetom v nej sú 2 body.



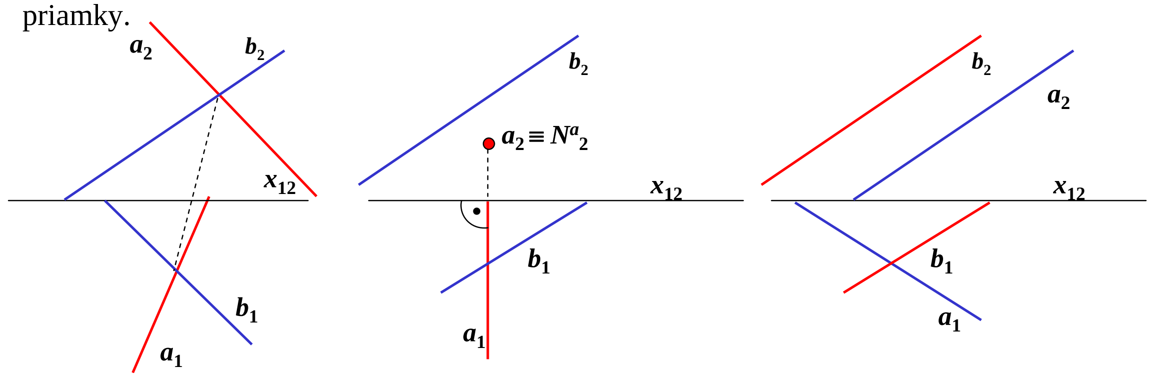


Vzájomná poloha 2 priamok v Mongeovej projekcii

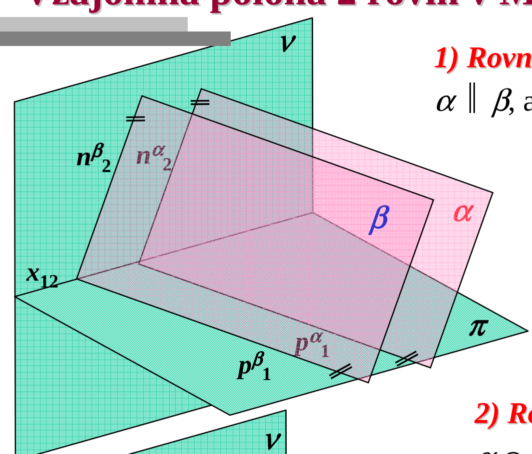
2) Rôznobežné priamky a, b: $a \cap b = R \Rightarrow a_1 \cap b_1 = R_1$, $a_2 \cap b_2 = R_2$, potom $R_1 R_2 \perp x_{12}$.



3) Mimobežné priamky a, b: neplatia predchádzajúce pravidlá pre rovnobežné, ani rôznobežné priamky.

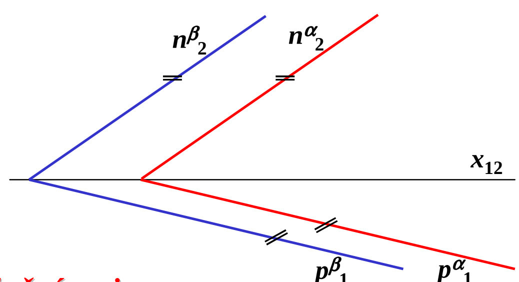


Vzájomná poloha 2 rovín v Mongeovej projekcii



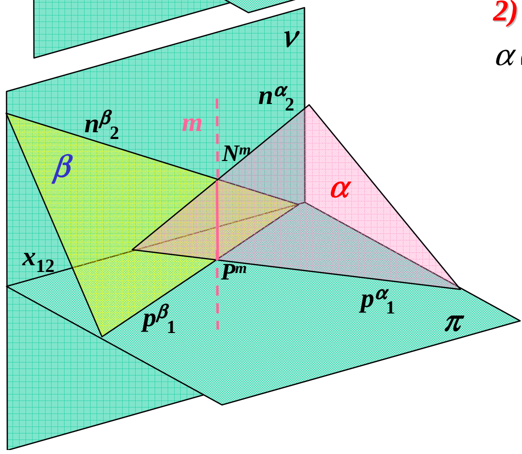
1) Rovnobežné roviny:

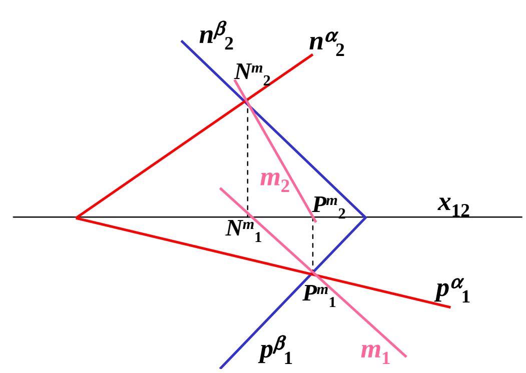
$\alpha \parallel \beta$, ak existujú ich stopy $\Rightarrow p^\alpha{}_1 \parallel p^\beta{}_1$, $n^\alpha{}_2 \parallel n^\beta{}_2$



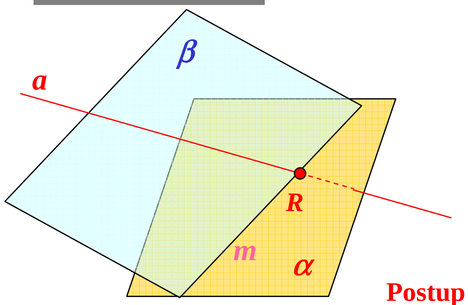
2) Rôznobežné roviny:

$\alpha \cap \beta = m \Rightarrow P^m = p^\alpha \cap p^\beta$, $N^m = n^\alpha \cap n^\beta{}_2$.





Vzájomná poloha priamky a roviny v Mongeovej projekcii



Všeobecný postup $a \cap \alpha$:

1. **Priamkou a preložíme ľubovoľnú rovinu β: $a \subset \beta$.**
2. **Nech m je priesečnica rovín α a β: $\alpha \cap \beta = m$.**
3. **Podľa vzájomnej polohy priamok a a m určíme vzájomnú polohu priamky a a roviny α:**

   a, $a \equiv m \Rightarrow a \subset \alpha$

   b, $a \parallel m \Rightarrow a \parallel \alpha$

   c, $a \cap m = R \Rightarrow R = a \cap \alpha$

Postup v Mongeovej projekcii, dané je $a[a_1, a_1]$, $\alpha(p^\alpha, n^\alpha)$, určte $a \cap \alpha$:

1. β: $a \subset \beta$, $\beta \perp \pi$

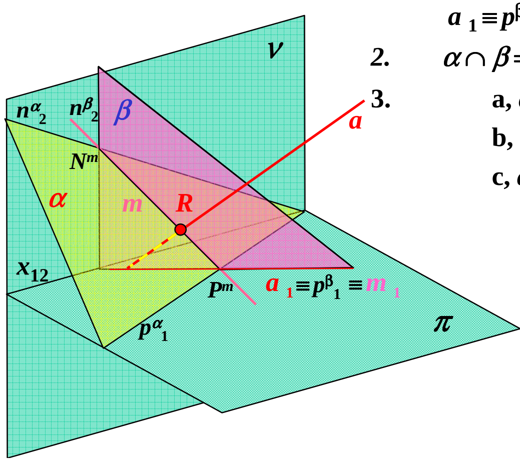
$a_1 \equiv p^\beta_1$, $n^\beta_2 \perp x_{12}$

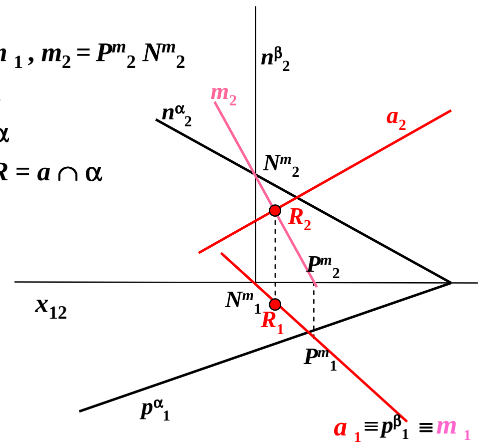
2. $\alpha \cap \beta = m$: $a_1 \equiv p^\beta_1 \equiv m_1$, $m_2 = P^m_2 N^m_2$

3. a, $a_2 \equiv m_2 \Rightarrow a \subset \alpha$

b, $a_2 \parallel m_2 \Rightarrow a \parallel \alpha$

c, $a_2 \cap m_2 = R_2 \Rightarrow R = a \cap \alpha$





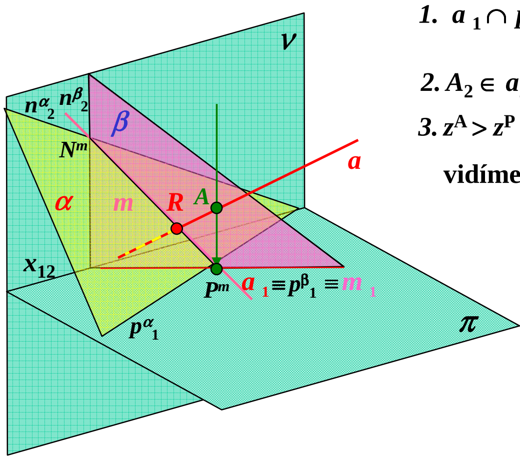
Viditeľnosť priamky vzhľadom na rovinu v Mongeovej projekcii

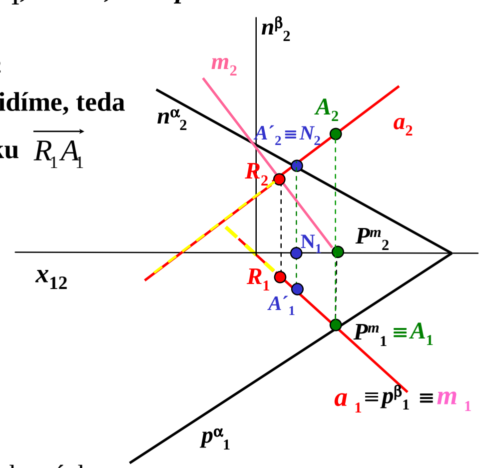
Viditeľnosť pôdorysu: Porovnávame bod na priamke a v rovine, ktorých pôdorysy sú totožné a viditeľný je ten, ktorý má väčšiu z-ovú súradnicu:

1. $a_1 \cap p^\alpha_1 = A_1 \equiv P^\alpha_1$, $A \in a$, $P^\alpha \in p^\alpha$

2. $A_2 \in a_2$, $P^\alpha_2 \in x_{12}$

3. $z^A > z^P \Rightarrow$ **bod A vidíme, teda vidíme polpriamku** $\overrightarrow{R_1 A_1}$





Viditeľnosť nárysu: Porovnávame bod na priamke a v rovine, ktorých nárysy sú totožné a viditeľný je ten, ktorý má väčšiu y-ovú súradnicu:

1. $a_2 \cap n^\alpha_2 = A'_2 \equiv N_2$, $A' \in a$, $N \in n^\alpha$,

2. $A'_1 \in a_1$, $N_1 \in x_{12}$,

3. $y^{A'} > z^N \Rightarrow$ **bod A vidíme, teda vidíme polpriamku** $\overrightarrow{R_2 A_2}$.