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B. SAMIA, F. ALI, A. FOUZI

NUMERICAL INTEGRATION OF ABRUPTLY UNSTEADY FLOWS IN AN OPEN CHANNEL

Boudjellal SAMIA
email: boudjellalsamia@yahoo.fr

Research field: fluid mechanics and hydraulics

Pr. Fourar ALI
email: fourarali@yahoo.fr

Research field: fluid mechanics and hydraulics

Abdessemed FOUZI
email: fouzi.abdessemed@hotmail.fr

Research field: fluid mechanics and hydraulics

Address:
Department of Hydraulics – Institute of Civil Engineering, Hydraulics and Architecture
Batna University
Street of Chahid Boukhlouf Med El Hadi
CP 05000 – Batna, Algeria.

ABSTRACT

Unsteady flows in channels and rivers combine a number of very complex natural phenomena, particularly phenomena related to the wave propagation caused by gravity or floods. The free surface flows relative to flows are regulated by the dynamic equilibrium of the forces of gravity, inertia and friction (Hervouet, 2003).In this present research study the focus is primarily on the dynamic behavior of flows in an open channel, which occurs after the opening of a partition in the case of an enlargement and finally an abrupt drop. The numerical integration of the governing hyperbolic partial differential equations allowed us to better understand the phenomenon studied in the three cases cited above. Solving these equations could be realized by using a FORTRAN calculation program that we developed.

KEY WORDS

- *Abruptly unsteady flows,*
- *Spillway,*
- *Divergent,*
- *Hyperbolic PDE,*
- *ADI scheme.*

1. INTRODUCTION

Understanding and predicting the behavior of free surface flows attracts a great deal of scientific interest. The design of protective structures against floods and channel diversions and evaluations of the impact of dam failures are topics, among many others, which crucially require a good understanding of the phenomena of hydraulics and fluid mechanics. Approaches for control are developed through a study of these phenomena using models, including physical models. With the intensive development of computational resources, we have gradually given ground to the use of numerical modeling.

Abruptly unsteady flows that occur during the sudden opening of a valve in the case of a diverging or elevated spillway are characterized by a sudden elevation and by marked agitation and great energy losses. These flows can occur in different ways according to the Froude number. Hydraulic characteristics begin to differ very slightly; then small eddies of varying sizes and frequencies are generated. This phenomenon of pulsating and high turbulence acquires its maximum value near the bottom or on the surface. The pulsations give rise to waves of irregular periodicity, which can propagate over long distances. Their wake affects transports of various types, because of constraints on the substance carried by these flows at very energetic and very high speeds.

With open flows, the main challenge is to determine the characteristics of the free surface: its position, shape and speed. This study will focus on transient free surface flows in a constant rectangular channel. The transient phenomena are systematically present because of the inherent variability of flows. They result in a variation which can be brutal in some cases and affect the water level due to the propagation of a gravity wave or a hydraulic jump. Indeed, when starting a pump or opening a valve, a wave can spread rapidly in prismatic and non-prismatic systems.

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2. MODELING OF TRANSIENT PHENOMENA IN OPEN CHANNELS

Barre of Saint-Venant equations are commonly used to model unsteady flows that gradually or abruptly vary on a free surface (Thual, 2003). The existence of nonlinear terms in these equations requires us to make some assumptions and approximations.

3. EXTREME CONDITIONS

3.1. Impermeability of the surface and the bottom

We assume that there is no mass transfer of water through the bottom and the surface, so we can write:

$$\frac{\partial Z_S}{\partial t}+U^S\frac{\partial Z_s}{\partial x}+V^s\frac{\partial Z_s}{\partial y}-W^s=0 \quad (1)$$

And:

$$\frac{\partial Z_f}{\partial t}+U^f\frac{\partial Z_f}{\partial x}+V^f\frac{\partial Z_f}{\partial y}-W^f=0 \quad (2)$$

3.2. Average of the Navier-Stokes equations

The Navier-Stokes equations with a constant density and hydrostatic pressure are averaged over the vertical integration from the bottom up to the surface.

3.2.1. Average of the continuity equation

The continuity equation:

$$\int_{Z_f}^{Z_s}\left(\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}+\frac{\partial W}{\partial z}\right)dz=0 \quad (3)$$

Using the Leibniz rule:

$$\int_{Z_f}^{Z_s}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)dz=\frac{\partial}{\partial x}\int_{Z_f}^{Z_s}Udz-U(x,y,Z_s)\frac{\partial Z_s}{\partial x}+U(x,y,Z_f)\frac{\partial Z_f}{\partial x} \quad (4)$$

$$\frac{\partial}{\partial y}\int_{Z_f}^{Z_s}Vdz-V(x,y,Z_s)\frac{\partial Z_s}{\partial x}+V(x,y,Z_f)\frac{\partial Z_f}{\partial Y}+W(Z_s)-W(Z_f) \quad (5)$$

Given the conditions of impermeability:

$$\frac{\partial}{\partial x}(hu)+\frac{\partial}{\partial y}(hv)+\frac{\partial Z_s}{\partial t}-\frac{\partial Z_f}{\partial t}=0 \quad (6)$$

Where:

$$\frac{\partial h}{\partial t}+div(h\vec{u})=0 \quad (7)$$

3.2.2. Average of the momentum equation

$$\frac{\partial(\vec{U})}{\partial t}+\vec{U}\nabla\vec{U}=-\frac{1}{\rho}\overrightarrow{grad}(p)+\frac{1}{\rho}div(\underline{\underline{\sigma}})+\vec{g}+\vec{F} \quad (8)$$

Apply the hydrostatic pressure assumption by writing that $p = \rho g(Z_s - z)$.

3.3. Pressure gradient

For pressure gradients in x and y, we write:

$$\int_{Z_f}^{Z_s}-\frac{1}{\rho}\frac{\partial}{\partial x}[\rho g(Z_s-z)]dz=-hg\frac{\partial Z_s}{\partial x} \quad (9)$$

And:

$$\int_{Z_f}^{Z_s}-\frac{1}{\rho}\frac{\partial}{\partial y}[\rho g(Z_s-z)]dz=-hg\frac{\partial Z_s}{\partial y} \quad (10)$$

The sum of all these terms gives the following equations (due to the impermeability of the bottom and free surface):

$$\frac{\partial(hu)}{\partial t}+\frac{\partial}{\partial x}(huu)+\frac{\partial}{\partial y}(huv)=-gh\frac{\partial Z_s}{\partial x}+hF_x+div\left(h\nu_e\overrightarrow{grad}(u)\right) \quad (11)$$

$$\frac{\partial(hv)}{\partial t}+\frac{\partial}{\partial x}(huv)+\frac{\partial}{\partial y}(hvv)=-gh\frac{\partial Z_s}{\partial y}+hF_y+div\left(h\nu_e\overrightarrow{grad}(v)\right), \quad (12)$$

Where F_x and F_y are the forces due to the wind and the Coriolis forces, which are negligible.
So:

$$\frac{\partial h}{\partial t}+div(h\vec{u})=0 \quad (13)$$

$$\frac{\partial(hu)}{\partial t}+\frac{\partial}{\partial x}(huu)+\frac{\partial}{\partial y}(huv)=-gh\frac{\partial Z_s}{\partial x}+div\left(h\nu_e\overrightarrow{grad}(u)\right) \quad (14)$$

$$\frac{\partial(hv)}{\partial t}+\frac{\partial}{\partial x}(huv)+\frac{\partial}{\partial y}(hvv)=-gh\frac{\partial Z_s}{\partial y}+div\left(h\nu_e\overrightarrow{grad}(v)\right) \quad (15)$$

Where t is the time; (x, y) are the Cartesian coordinates of a point on the horizontal plane; h (x, y, t) is the water depth; *Zs* (x, y, t) is the bottom elevation; u (x, y, t) is the component of the velocity vector along the axis of the abscissa; and v (x, y, t) is its component along the axis of the ordinates,

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ν – The kinematic viscosity of the fluid and g is the acceleration vector norm.
ρ – Density of the fluid.
These equations are the two-dimensional Saint-Venant equations. Mathematical modeling was performed on the transient free surface flows in a rectangular channel caused by the sudden opening of a valve, the existence of a divergence (sudden enlargement), and a spillway, which was placed in the upstream extremity of the channel, there by producing a change in the water depth and speed variations over time in different sections of the canal; the evolution of these depths may cause flooding of the canal. The application of the conservation laws of mass and momentum in this model resulted in a system of two partial differential hyperbolic equations, which were solved numerically using a finite volume numerical scheme (ADI) (Audusse, 2004 ; Chorin, 1968; Fennema et al., 1989).

4. INTERPRETATIONS

A numerical simulation of the transient free surface caused by the occurrence of the sudden opening of a valve, a divergence, and a spillway was developed. The application of the conservation laws of mass and momentum led to a system of two partial differential hyperbolic equations, which was solved numerically using the ADI scheme. The transient flow with a free surface concerns a rectangular channel with a zero slope and a smooth bottom. The transient flow allowed us to study the evolution of the variations of the depth and velocity fields over time in different sections of the channel. We saw changes in the profiles of the depths and velocities and the appearance of oscillations during the evolution of these depths, which may cause flooding of the canal.

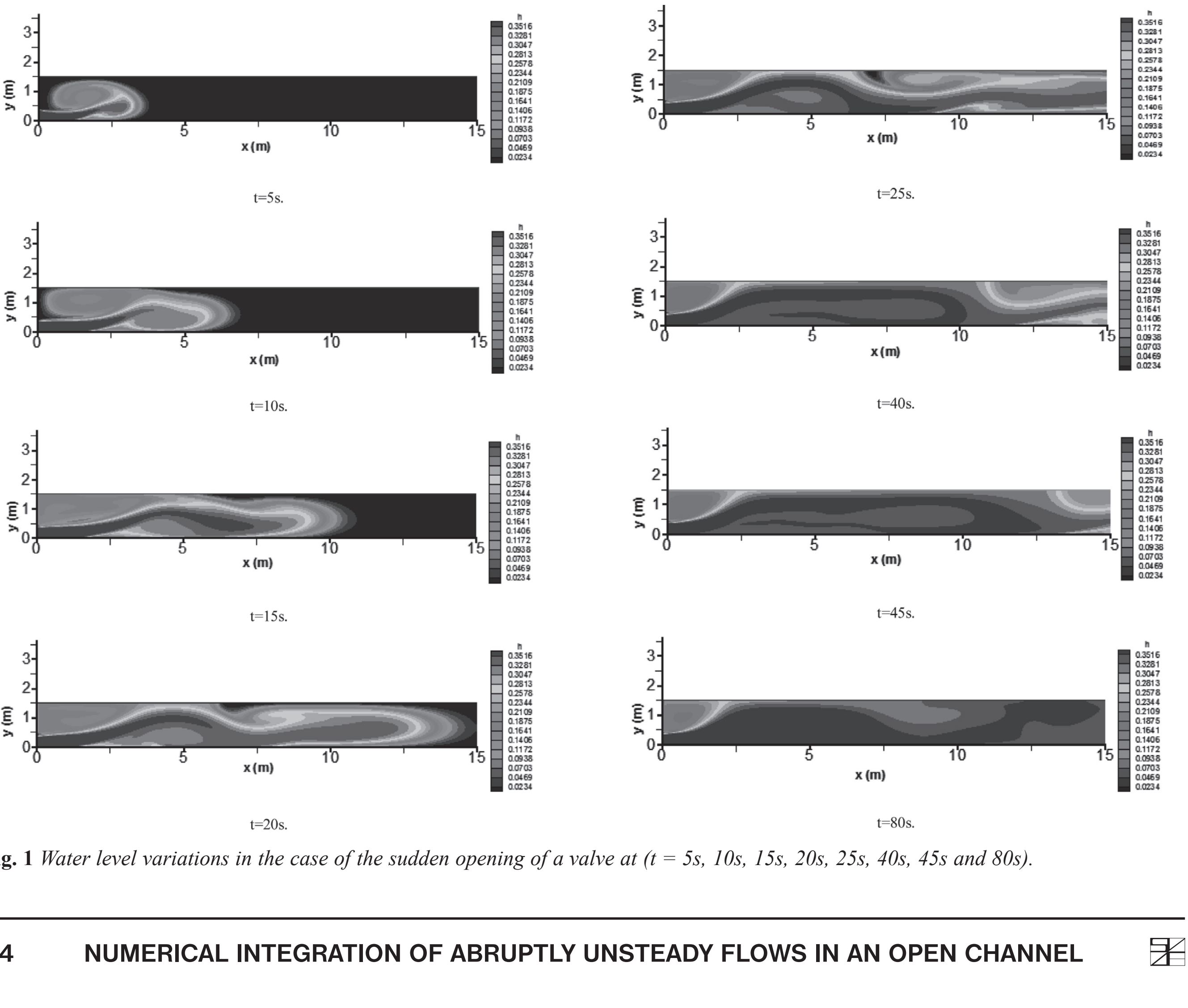

Fig. 1 *Water level variations in the case of the sudden opening of a valve at (t = 5s, 10s, 15s, 20s, 25s, 40s, 45s and 80s).*

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4.1. Sudden opening of a valve in a channel with a zero slope and friction

The first case concerns a transient flow in a prismatic channel with a rectangular section. We consider a horizontal channel of a width of B = 0.50m, a height of H = 1.50m and a length of L = 15m. The bed of the channel is thoroughly smooth, so there was no effect from friction on the flow. At the beginning of time t = 0, the valve is fully open. The water released on the abrupt opening of the valve moves downstream in the form of a wave height of 1.5 meters and easily reaches the distance between 5 and 10 m of the valve. The numerical discretization is done by dividing the channel into (200x100) computing nodes. The following results show the variations of the height of the wave and its velocity.

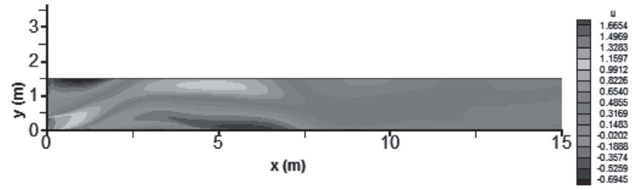

Fig. 2 *Variation of the velocity u (m/s) in the case of the sudden opening of the valve.*

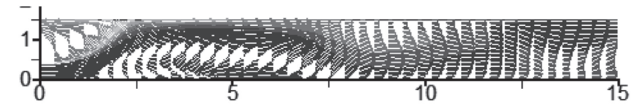

Fig. 3 *Presentation of the fields of the velocity vectors in the case of the sudden opening of the valve.*

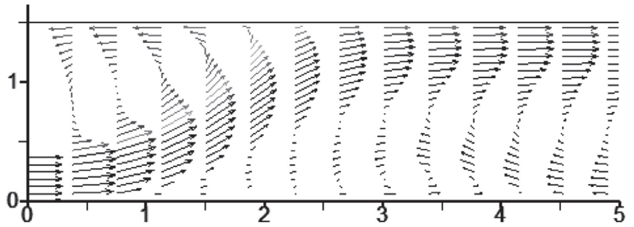

Fig. 4 *Presentation of the recirculation zone.*

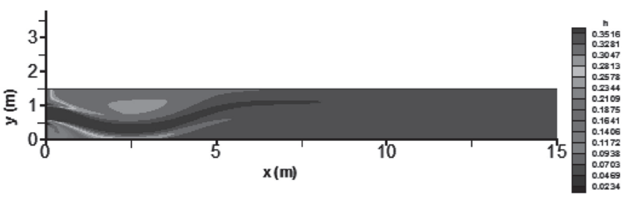

Fig. 5 *Water level variations in the case of a divergence.*

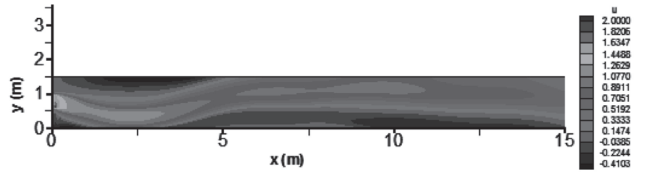

Fig. 6 *Velocity variation u (m/s) in the case of a divergence.*

Fig. 7 *Presentation of velocity vector fields in the case of a divergence.*

The second case concerns the flow through a divergence located at the entrance of the canal.

In the brutal divergence the velocity gradients are very strong at the edges. Significant turbulence takes shape along with relatively low speeds in the area near the downstream wall of the divergence, and there is a high degree of pressure in the same area.

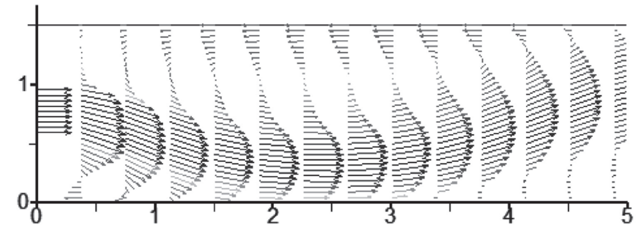

Fig. 8 *Presentation of the recirculation zone.*

4.2. The presence of a spillway at the entrance to a channel with a zero slope and without any friction

The third case concerns the flow through a spillway.

5. RESULTS AND DISCUSSION

A hydraulic system is often used for storage and flow regulation. The restitution of the waters downstream of these structures can

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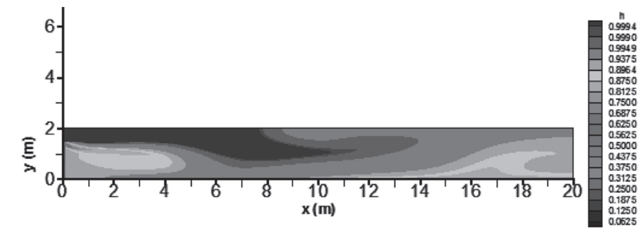

Fig. 9 *Water level variations in the case of a spillway at t = 50 s.*

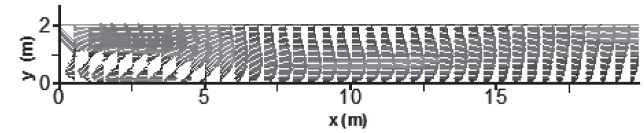

Fig. 10 *Presentation of the recirculation zone.*

sometimes be made with great velocities, whose impacts on the bottom are very damaging. The realization of damper basins to break the devastating effects of the released water is more forceful than necessary. Within these works, the hydraulic jump caused by using a threshold is the seat of the dissipation of the energy released. The role assigned to these types of pools is to promote the formation of a hydraulic jump and increase energy dissipation, which makes the sink more compact and improves the stability of the process. Many basin configurations may be envisaged, but this case dealt with a prismatic horizontal channel with a rectangular section, inside which a threshold was placed.

This study, the objective of which was to try to better understand a suddenly unsteady flow as a complex hydraulic phenomenon, was conducted by simulating a channel 15 m long, 0.5 m wide and 1.5m high; the suddenly unsteady flows studied are those generated by the sudden opening of a valve in the case of a divergence or a spillway. The velocity distribution shows areas with rolls of the surface and bottom, where the rates are negative. The area of the bottom of the roll behind the threshold is characterized by a recirculation of the water causing a dissipation of the energy.

6. CONCLUSION

A suddenly varied unsteady flow in a rectangular channel with a free surface is one of the least studied and more complex phenomena in the area of the free surface of water. In the present study, we were particularly interested in the phenomenon of hydraulic jumps induced by opening a valve, the expansion of the canal and then a sharp drop from a weir. These phenomena are very useful in determining the relationship between the depths combined just before and after the formation of projections; the losses in cases of different projections are also very important and sometimes reach enormous values, as a study of the longitudinal profile of water in canals and rivers shows. The equations describing these phenomena involving both external forces are equations of momentum.

The numerical integration of the partial differential equations (PDEs) of Saint-Venant 2D has allowed us to better design the phenomenon of a suddenly varied unsteady surface. The resolution of this system of hyperbolic partial differential equations was performed using the finite volume method (ADI scheme), which is a robust digital system. It is capable of reducing the constraints on the time step and space, facilitates the convergence of the calculation scheme and gives a numerical solution, which reproduces as much as possible the reality of an unsteady free surface. After fixing the boundary conditions on the walls and the free surface into and out of the field, a Fortran IV computer program was developed. The results obtained through this program are very successful and converge perfectly with the theory.

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