Ing. Gabriela Pavlendová, PhD.

FUNDAMENTAL PHYSICS

EXAMPLES

SLOVENSKÁ TECHNICKÁ
UNIVERZITA V BRATISLAVE

SLOVAK UNIVERSITY OF TECHNOLOGY IN BRATISLAVA
FACULTY OF CIVIL ENGINEERING

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1 INTRODUCTION

Science is beautiful when it makes simple explanations of phenomena or connections between different observations. Examples include the double helix in biology and the fundamental equations of physics. *Stephen Hawking*

There is no science in this world like physics. Nothing comes close to the precision with which physics enables you to understand the world around you. It's the laws of physics that allow us to say exactly what time the sun is going to rise. What time the eclipse is going to begin. What time the eclipse is going to end. *Neil deGrasse Tyson*

It is wrong to think that the task of physics is to find out how Nature is. Physics concerns what we say about Nature. *Niels Bohr*

Learn from yesterday, live for today, hope for tomorrow. The important thing is to not stop questioning. *Albert Einstein*
In the beginning there was nothing, which exploded. *Terry Pratchett*

Physics is the science from life and for life. The more you study the more you get intrigued. I wish the readers to become fascinated by the wonders Physics work with. *The author*

I would like to express my deepest thanks to prof. RNDr. Ivan Baník, PhD. and assoc. prof. RNDr. František Čulík, PhD, who have very kindly read and reread the manuscript and helped me with their remarks and advices. The head of our department assoc. prof. RNDr. Jozefa Lukovičová PhD was supporting me with whatever was needed, many thanks for that. My cordial thanks belong to all my children (Pavel, Gabriela, Lucia, Jana and Ján) my daughter in law Beata, my grandsons Tomáš and David, my son in law Juraj and my mother for their understanding me being busy for such a long time. But above all, there was the help and the support of my husband, there are no words I could express how thankful I am.

2 SCALARS AND VECTORS

In physics, there are quantities that are described by a single number, for example, the mass of a person. Such quantities are called scalars. For others we need more than one number – these are e.g. vectors. A vector quantity is one that has both a magnitude and a direction. Examples of vectors are displacement, velocity, acceleration, force, and linear momentum. A vector may be represented geometrically by an arrow of length proportional to its magnitude, pointing in the assigned direction. For graphical representation we use Cartesian coordinate system. In this textbook we will use $\boldsymbol{a}$ (bold, italic) to asign a vector and a to assign the magnitude of a vector.

ADDING VECTORS

When you add scalars, five and four, for example, then there is only one answer that is nine. When you have two vectors and no information on their direction, but you know that the magnitude of one is four and the magnitude of the other is five, then the magnitude of the sum could be nine, if they are both in the same direction or it could be one, if they are in opposite directions (these are the extremes). And you have the whole range of possibilities if you do not know the direction. So adding vectors is more complicated. Choose any of the vectors to be summed and draw it as an arrow in the correct direction and of the correct length- remember to put an arrowhead on the end to denote its direction. Take the next vector and draw it as an arrow starting from the arrowhead of the first vector in the correct direction and of the correct length. Continue until you have drawn each vector- each time starting from the head of the previous vector. In this way, the vectors to be added are drawn one after the other tail-to-head. Therefore this is sometimes called **Tail-to-head method** for adding vectors (any number of vectors). The resultant is then the vector drawn from the tail of the first vector to the head of the last. Its magnitude can be determined from the length of its arrow using the scale. Its direction can be determined from the scale diagram, too. Another method is sometimes called **parallelogram**. In the parallelogram method for vector addition, the vectors are translated (i.e., moved) to a common origin and the parallelogram is constructed.

E.g. $\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{c}$

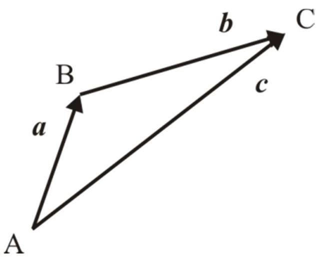


Fig.2.1 Tail to head method

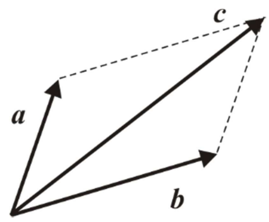


Fig.2.2 Parallelogram

UNIT VECTORS

Vector quantities are often expressed in terms of unit vectors. A unit vector is a dimensionless vector having a magnitude of exactly 1. Unit vectors are used to specify a given direction and have no other physical significance. We use the symbols $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ to represent unit vectors pointing in the positive *x*, *y*, and *z* directions of the Cartesian coordinate system and so the unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ form a set of mutually perpendicular vectors.

Unit vector of any vector is usually signed with zero index $\boldsymbol{a}^{\boldsymbol{0}}$ and so the vector $\boldsymbol{a}$ with magnitude *a* can be written in the following way:

$$\boldsymbol{a} = a\boldsymbol{a}^{\boldsymbol{0}}$$

THE VECTOR COMPONENTS

In two dimensions the vector components are as follows:

$$a_x = a\cos\alpha, \quad a_y = a\sin\alpha$$

$$\boldsymbol{a} = (a_x, a_y) = a_x\boldsymbol{i} + a_y\boldsymbol{j}$$

The magnitude *a* of vector $\boldsymbol{a}$:

$$a = |\boldsymbol{a}| = \sqrt{{a_x}^2 + {a_y}^2} \quad \text{and} \quad \tan\alpha = a_y/\,a_x$$

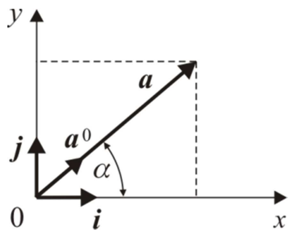


Fig.2.3 Vector components.

In three dimensions: $\boldsymbol{a} = (a_x, a_y, a_z) = a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k}$

$$a = |\boldsymbol{a}| = \sqrt{{a_x}^2 + {a_y}^2 + {a_z}^2}$$

VECTOR MULTIPLICATION

There are several ways of multiplying vectors. The reason for this diversity is that in forming the "product" of two vectors, we must take into account both their magnitudes and their directions. Depending on how we combine these quantities, we obtain different kinds of products. The two most important kinds of products are the scalar product and the vector product.

THE SCALAR PRODUCT OF VECTORS (also called the dot product) is the scalar defined as

$$\boldsymbol{a} \cdot \boldsymbol{b} = ab\cos\varphi = a_x b_x + a_y b_y + a_z b_z$$

φ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$, and it is between 0° and 180°, and a, b are the magnitudes of the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

The scalar product of two vectors $\boldsymbol{a}$

$$\boldsymbol{a} \cdot \boldsymbol{a} = aa\cos 0 = a^2 = a_x^2 + a_y^2 + a_z^2$$

Interesting is the scalar product of two unit vectors $\boldsymbol{a}^{\mathbf{0}}$**.**

Assigning angles between vector $\boldsymbol{a}$ and vectors $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ as α, β, γ we can write vector $\boldsymbol{a}$ and unit vector $\boldsymbol{a}^{\mathbf{0}}$ as follows: $\boldsymbol{a} = a(\boldsymbol{i}\cos\alpha + \boldsymbol{j}\cos\beta + \boldsymbol{k}\cos\gamma)$**,** $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are called direction cosines.

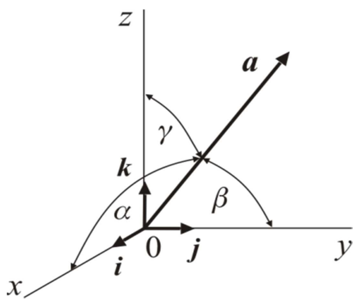


Fig.2.4 Vector components.

$$\boldsymbol{a}^{\mathbf{0}} \cdot \boldsymbol{a}^{\mathbf{0}} = (\boldsymbol{i}\cos\alpha + \boldsymbol{j}\cos\beta + \boldsymbol{k}\cos\gamma) \cdot (\boldsymbol{i}\cos\alpha + \boldsymbol{j}\cos\beta + \boldsymbol{k}\cos\gamma)$$
$$= \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

THE VECTOR PRODUCT (also called the cross product), defined as

$$\boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{b} = -\boldsymbol{b} \times \boldsymbol{a} = (ab \sin \varphi)\boldsymbol{c^0}$$

where a and b are vectors' magnitudes of vectors ***a, b,*** and φ is angle between these vectors ($\varphi \leq 180°$). The resultant of vector product is a vector with magnitude given in the brackets.
The vector $\boldsymbol{c}$ is perpendicular to the plane of the vectors $\boldsymbol{a}$ and ***b.*** Its direction is in the direction of the unit vector $\boldsymbol{c^0}$, given by the right-hand rule or right-handed screw.

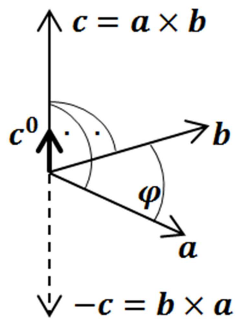


Fig.2.5 The vector product

EXAMPLE 2.1 A woman walks 250 m in the direction 30º east of north, then 175 m directly east.
(a) Using graphical methods, find her final displacement from the starting point.
(b) Compare the magnitude of her displacement with the distance she walked.

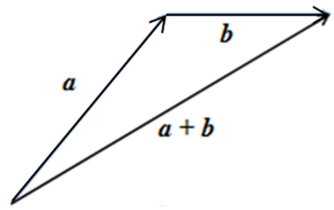


Fig.2.6 Adding vectors

Solution (a) Draw the first vector ***a*** with a length, to scale, of 250 m, 30º east of north and the second vector ***b*** with a length, to the same scale, of 175 m, due east. The order is not important but the tail of one vector must be at the head of the other. Now draw the resultant vector ***r***, from the tail of the first to the head of the second. Use a rule to measure its length and use the scale to find its magnitude. Use a protractor to measure the angle it makes with one of the cardinal compass directions. The magnitude of the resultant is ∼370 m and the resultant is 36º north of east.
(b) The magnitude of her displacement is 370 m; the distance she walked is 250 m + 175 m = 425 m.

EXAMPLE 2.2 (a) What is the sum in unit-vector notation of the two vectors ***a*** = 4***i*** + 3***j*** and ***b*** = -13***i*** + 7***j*?** (b) What is the magnitude and direction of ***a*** + ***b***?
Solution
(a) Let ***r = a + b.*** Then. $r_x = a_x + b_x = 4 - 13 = -9$ and $r_y = a_y + b_y = 3 + 7 = 10$.
Thus ***r*** = -9***i*** + 10***j***.

Or $\boldsymbol{r} = \boldsymbol{a} + \boldsymbol{b} = (4\boldsymbol{i} + 3\boldsymbol{j}) + (-13\boldsymbol{i} + 7\boldsymbol{j}) = -9\boldsymbol{i} + 10\boldsymbol{j}$

(b) The magnitude of the resultant is $r = \sqrt{{r_x}^2 + {r_y}^2} = \sqrt{(-9)^2 + 10^2} = 13.45$

The angle θ between the resultant and the positive x axis is given by:

$\tan\theta = r_y/r_x = -10/9 = -1{,}1.$ So θ is either -48º or 132º. The first angle has a positive cosine and a negative sine while the second angle has a negative cosine and positive sine. Since the x component of the resultant is negative and the y component is positive, θ = 132º. Another approach: the resultant lies in the upper half-plane, so θ = 132º.

EXAMPLE 2.3 A boat crossing a wide river moves with a speed of 10 km/h relative to the water. The water in the river has a uniform speed of 5 km/h due east relative to the Earth.

(a) If the boat heads due north, determine the velocity and direction of the boat relative to an observer standing on either bank. (11.2 km/h, 26.6 ° east of north).

(b) If the boat travels with the same speed of 10 km/h relative to the river and is to travel due north, what should its heading be, what will the velocity of the boat relative to an observer standing on either bank be? (30° west of north, 8.66 km/h).

EXAMPLE 2.4 A car travels 20.0 km due north and then 35.0 km in a direction 60.0° west of north. Find the magnitude and direction of the car's resultant displacement. (48.2 km, 38.9°)

EXAMPLE 2.5 Three vectors, ***A*** with angle 28° with respect to (+) x axis, ***B*** with the angle 56° with respect to (-) x axis and ***C*** in the direction of (-) y axis. Magnitudes A= 44.0, B= 26,5 and C= 31.0 are given in arbitrary units. Determine the sum of the three vectors. Give the resultant in terms of (a) components ($(A+B+C)_x$ = 24; $(A+B+C)_y$= 11.6) (b) magnitude and angle with the x axis (26.7, 25.8°)

EXAMPLE 2.6 The magnitude of ***a*** is 2 and the magnitude of ***b*** is 8. What is the angle between the vectors ***a, b*** if $|\boldsymbol{a} \times \boldsymbol{b}| = 16$? (90°)

EXAMPLE 2.7 The magnitude of ***a*** is 2 and the magnitude of ***b*** is 8 and the angle between the vectors ***a*** and ***b*** is 90°. Determine the magnitude of $\boldsymbol{a} \cdot \boldsymbol{b}$. (0)

3 KINEMATICS

Kinematics is the study of motion. To describe motion, we must refer it to the frame of reference. We often use a reference frame attached to the surface of the earth or to the room floor. The basic concepts involve understanding the meaning of displacement, velocity and acceleration. These physical quantities are vectors; however considering the motion in one dimension we are able to describe vector properties simply by assigning a plus or minus sign to them.

Summary of key equations for motion in one dimension – linear motion.

Average velocity: $\overline{v} = \frac{x_2 - x_1}{t_2 - t_1}$ Average acceleration: $\overline{a} = \frac{v_2 - v_1}{t_2 - t_1}$

Instantaneous velocity: $v = \frac{\mathrm{d}x}{\mathrm{d}t}$ Instantaneous acceleration: $a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2x}{\mathrm{d}t^2}$

If a = constant, then:

$v = v_0 + at$

$x = x_0 + v_0 t + at^2/2$

$v^2 = v_0^2 + 2ax$

If $a = 0$, then

$v = v_0$ (constant)

$x = x_0 + vt$

For a freely falling object:

$a = -g$

$v = v_0 - gt$

$y = y_0 + v_0 t - gt^2/2$

An object dropped from rest will fall a distance h in time t where:

$h = gt^2/2$

Motion in plane or space.

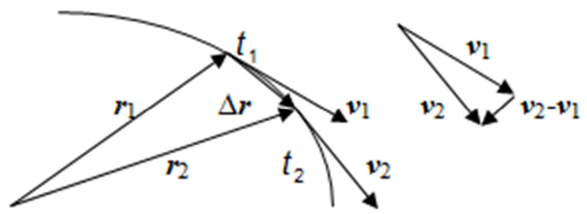


Fig.3.1 Motion in plane

For motion in plane or space we must use more explicit notation to make clear the vector properties. We specify the position of a particle by the **position vector *r*.** As the particle moves, ***r*** changes. If at time t_1 the position vector is $\boldsymbol{r}_1$ and velocity $\boldsymbol{v}_1$, and at time t_2 it is $\boldsymbol{r}_2$

and $\boldsymbol{v}_2$, the **displacement vector** $\Delta\boldsymbol{r}$ for this time interval is defined as $\Delta\boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_1$. The velocity vector is directed tangent to the path of the particle. The acceleration vector can be oriented in any direction, depending on what is happening.

The position, velocity, and acceleration vectors for a particle moving in the *x*-*y* plane are:

$$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j}$$

$$\mathbf{v} = \frac{\mathrm{d}}{\mathrm{d}t}r(t) = \frac{\mathrm{d}}{\mathrm{d}t}(x\boldsymbol{i} + y\boldsymbol{j}) = \frac{\mathrm{d}x}{\mathrm{d}t}\boldsymbol{i} + \frac{\mathrm{d}y}{\mathrm{d}t}\boldsymbol{j} = v_x\boldsymbol{i} + v_y\boldsymbol{j} = \boldsymbol{v_x} + \boldsymbol{v_y}$$

$$\boldsymbol{a} = \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{v}(t) = \frac{\mathrm{d}}{\mathrm{d}t}\left(v_x\boldsymbol{i} + v_y\boldsymbol{j}\right) = \frac{\mathrm{d}v_x}{\mathrm{d}t}\boldsymbol{i} + \frac{\mathrm{d}v_y}{\mathrm{d}t}\boldsymbol{j} = a_x\boldsymbol{i} + a_y\boldsymbol{j} = \boldsymbol{a_x} + \boldsymbol{a_y}$$

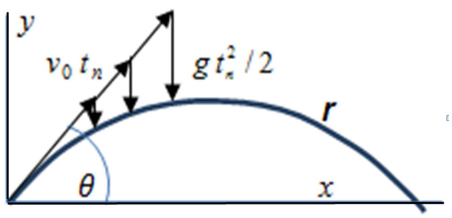
$$a_x = \frac{\mathrm{d}v_x}{\mathrm{d}t} = \frac{\mathrm{d}^2x}{\mathrm{d}t^2} \qquad\qquad a_y = \frac{\mathrm{d}v_y}{\mathrm{d}t} = \frac{\mathrm{d}^2y}{\mathrm{d}t^2}$$

The magnitudes of these vectors are:

$$r = |\boldsymbol{r}| = \sqrt{x^2 + y^2} \qquad v = |\boldsymbol{v}| = \sqrt{v_x^2 + v_y^2} \qquad a = |\boldsymbol{a}| = \sqrt{a_x^2 + a_y^2}$$

$x - \text{components}$	$y - \text{components}$
$a_x = \text{const}$	$a_y = \text{const}$
$v_x = v_{0x} + a_x t$	$v_y = v_{0y} + a_y t$
$x = x_0 + v_{0x}t + a_x t^2/2$	$y = y_0 + v_{0y}t + a_y t^2/2$
$v_x^2 = v_{0x}^2 + 2a_x x$	$v_y^2 = v_{0y}^2 + 2a_y y$

Projectile motion:



$$a_x = 0 \qquad a_y = -g$$

$$v_x = v_0\cos\theta = \text{const} \qquad v_y = v_0\sin\theta - gt$$

$$x = v_x t = v_0 t\cos\theta \qquad \mathrm{y} = \int v_y \mathrm{d}t = $$

$$= v_0 t\sin\theta - gt^2/2$$

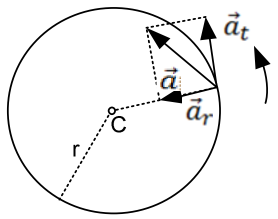
Fig. 3.2 Projectile motion

The position vector $\boldsymbol{r}$ of a projectile consists of two parts. The vector $\boldsymbol{v}_0 t$ would be the displacement of the projectile if gravity was absent, and the vector $gt^2/2$ is its vertical displacement due to its downward gravitational acceleration. So we can see that projectile motion is the superposition of two motions: (1) constant-velocity motion $\boldsymbol{v}_0 t$ in the direction of the elevation angle θ and (2) free-fall motion $gt^2/2$ in the vertical direction.

Circular motion - a point rotating in a plane about an axis perpendicular to this plane. Let us place the coordinate system beginning into the center of rotation. We describe the position of the point by the position vector $\boldsymbol{r}$ with magnitude r and angle θ. The point moves with peripheral speed $v = 2\pi r/T$ (T is period) and angular speed $\omega = \mathrm{d}\theta/\mathrm{d}t = v/r$ (in radians per second), and – by non-uniform circular motion – with angular acceleration $\alpha = \mathrm{d}\omega/\mathrm{d}t$. The component of the acceleration vector $\boldsymbol{a}$ that is parallel or antiparallel to $\boldsymbol{v}$ (that is, along the line tangent to the path) is called the **tangential acceleration $\boldsymbol{a}_\mathrm{t}$**. A more descriptive name is “speeding up or slowing down acceleration”. This is the kind of acceleration that measures changes in speed. The other component of acceleration, always perpendicular to $\boldsymbol{a}_\mathrm{t}$, is called **normal** or **radial acceleration $\boldsymbol{a}_\mathrm{r}$.** This radial acceleration $\boldsymbol{a}_\mathrm{r}$ has magnitude

$$a_\mathrm{r} = v^2/r = \omega v = \omega^2 r$$

An object undergoing uniform circular motion is moving with a constant speed. Nonetheless, it is accelerating due to its change in direction. The direction of the acceleration is inwards.

 *Fig.3.3Acceleration components of circular motion*	$\omega = \frac{2\pi}{T} = 2\pi f, \quad v = 2\pi r/T = \omega r$ $a_\mathrm{r} = v^2/r = \omega v = \omega^2 r$ $a_\mathrm{t} = \mathrm{d}v/\mathrm{d}t = \mathrm{d}(\omega r)/\mathrm{d}t = r\alpha$ $\boldsymbol{a} = \boldsymbol{a}_\mathrm{t} + \boldsymbol{a}_\mathrm{r}$ $a = \lvert\boldsymbol{a}\rvert = \sqrt{a_\mathrm{t}^2 + a_\mathrm{r}^2} = r\sqrt{\alpha^2 + \omega^4}$	If $\alpha = \text{const}$: $\omega = \omega_0 + \alpha t$ $\theta = \theta_0 + \omega_0 t + \alpha \frac{t^2}{2}$ $\omega^2 = \omega_0^2 + 2\alpha t$

Note the parallels between linear and circular motion equations.

EXAMPLE 3.1 An unidentified naval vessel is tracked by the Navistar Global Positioning System. With respect to a coordinate origin (0, 0) fixed at a lighthouse beacon, the position P of the vessel is found to be x_1 =2.0 km east, y_1 = 1.6 km north at t_1 = 0.30 h and x_2 = 6.4 km west, y_2 = 6.5 km north at t_2 = 0.60 h. Using west-east as the x axis and south-north as the y axis, determine the average velocity in terms of its components. What are the direction and magnitude of the average velocity in kilometers per hour?

Solution

$$\boldsymbol{v} = \frac{\boldsymbol{P}_2 - \boldsymbol{P}_1}{t_2 - t_1} = \frac{(-6.4\boldsymbol{i} + 6.5\boldsymbol{j}) - (2.0\boldsymbol{i} + 1.6\boldsymbol{j})}{0.6 - 0.3} = \frac{(-8.4\boldsymbol{i} + 4.9\boldsymbol{j})}{0.3}\,\mathrm{km/h}$$

$$v = \lvert\boldsymbol{v}\rvert = \sqrt{(-8.4)^2 + 4.9^2}/0.3 = 9.72/0.3 = 32.4\ \mathrm{km/h}$$

$$\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}(4.9/(-8.4)) = -30.3° \qquad 30.3° \text{ N of W}$$

EXAMPLE 3.2 A motorcycle accelerates quickly from rest, with an acceleration that has an initial value a_0 = 4.0 m/s^2 at t = 0 and decreases to zero during the interval $0 \le t \le 5.0$ s according to $a = a_0(1 - t/5\text{ s})$. After t = 5.0 s, the motorcycle maintains a constant velocity. What is this final velocity? In the process of "getting up to speed," how far does the motorcycle travel?

Solution

The acceleration a is given as an explicit function of time. Since the motorcycle accelerates from rest, the initial velocity is v_0 = 0, so v as a function of t is:

$$v = v_0 + \int_0^t a\,dt = 0 + \int_0^t a_0\left(1 - \frac{t}{5{,}0s}\right)dt = a_0\left(\int_0^t dt - \frac{1}{5{,}0s}\int_0^t t\,dt\right) = a_0\left(t - \frac{t^2}{10s}\right)\Bigg|_0^t = a_0\left(t - \frac{t^2}{10s}\right)$$

At t = 5.0 s, this velocity reaches its final value of 10 m/s.

To obtain the distance traveled during the acceleration, we must solve

$$x - x_0 = \int_0^t v\,dt = \int_0^t a_0\left(t - \frac{t^2}{10s}\right)dt = a_0\int_0^t\left(t - \frac{t^2}{10s}\right)dt = a_0\left(\frac{t^2}{2} - \frac{t^3}{30s}\right)\Bigg|_0^t = a_0\left(\frac{t^2}{2} - \frac{t^3}{30s}\right)$$

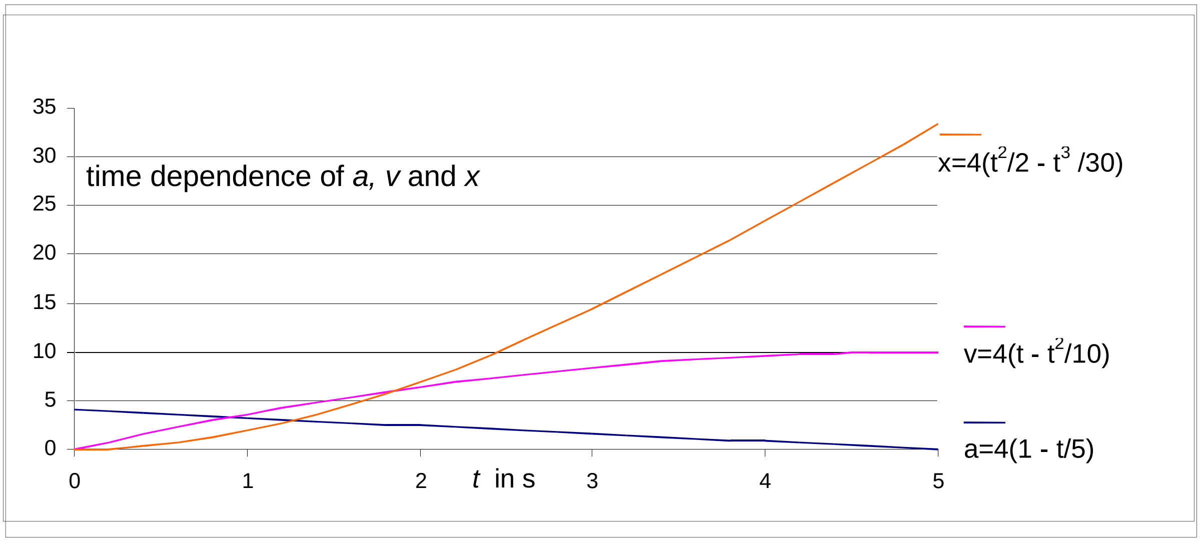


Fig. 3.4 Time dependence of a, v and x

Evaluating this expression at t = 5.0 s, we find $x - x_0 = 33$ m.

EXAMPLE 3.3 While standing in an elevator, you see a screw fall from the ceiling. The ceiling is 3 m above the floor. (a) If the elevator is moving upward with a constant speed of 2.2 m/s, how long does it take for the screw to hit the floor? (b) How long is the screw in the air if the elevator starts from rest when the screw falls, and moves upward with a constant acceleration of a_e = 4.0 m/s^2?

Solution

a) The position functions for the elevator floor is x_f and for screw x_s. Let us choose the origin to be the initial position of the floor, and designate upward as the positive direction.

$$x_f - x_{0f} = v_{0f}t + \frac{1}{2}a_f t^2 \qquad\qquad x_s - x_{0s} = v_{0s}t + \frac{1}{2}a_s t^2$$

Initial conditions and the accelerations

$x_{0f} = 0$ m, $v_{0f} = 2, 2$ m/s, $a_f = 0$ m/s^2 $\qquad$ $x_{0s} = h = 3$m, $v_{0s} = 2, 2$ m/s, $a_s = -g$

Substituting these values into the position functions we get:

$x_f = (2.2\text{ m/s})t$ $\qquad$ $x_s = h + (2.2\text{ m/s})t - (1/2)gt^2$

Set $x_f = x_s$ and solving for t gives: $\qquad t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\cdot 3\text{ m}}{9.81\text{m}\cdot\text{s}^{-2}}} = 0.78\text{ s}$

b) The elevator floor moves upward from rest with constant acceleration. The initial conditions are then:

$x_{0f} = 0$, $v_{0f} = 0$ m/s, $a_f = 4{,}0$m/s^2 $\qquad$ $x_{0s} = h = 3$ m, $v_{0s} = 0$ m/s, $a_f = -g$

Using the initial conditions to write the position functions for this case:

$$x_f = (1/2)a_f t^2 \qquad\qquad x_s = h - (1/2)gt^2$$

Now substituting the values and setting $\quad x_f = x_s$ we get the value of t

$$(1/2)a_f t^2 = \text{h} - (1/2)\text{g}\,t^2 \qquad\qquad t = \sqrt{\frac{2h}{g+a_f}} = 0.66$$

Remark: The time in the air is independent of the speed of the elevator, as long as the elevator does not accelerate. If the elevator has acceleration a_f you and the screw experience an "perceived gravity" with acceleration $g' = g + a_f$. For the case in which the elevator accelerates downward with $a_f = -g$, the time of fall becomes infinite and the screw appears weightless.

EXAMPLE 3.4 A ball is thrown straight up. Show that it spends as much time rising as it does falling back to its starting point.

Solution

At the peak of its flight $v = 0$. Thus $v = v_0 - gt_1 = 0$.

Rise time is $t_1 = v_0/g$

Elevation is given by $\qquad y = v_0 t - 1/2\ gt^2 \qquad$ assuming $\qquad y_0 = 0$.

When the ball returns to its starting point, $y = 0$.

Thus $\quad y = 0 = v_0 t — 1/2\ gt^2$, or

$t = 2v_0/g = 2t_1$. The total time in the air is twice the rise time, so fall time = rise time.

EXAMPLE 3.5 A ball is thrown upward with speed 12 m/s from the top of a building. How much later must a second ball be dropped from the same starting point if it is to hit the ground at the same time as the first ball? The initial position is 24 m above the ground.

Solution

The time for the first ball to reach the ground is t_1, $y = y_0 + v_o t - (1/2)\, g t^2$. Let $y = 0$ at starting point, so $y = -h = -24$ m at the ground.

$v_0 = 12$m/s

$-24 = 0 + 12t - (1/2)(9.8)\, t^2$ $\quad 4.9t^2 - 12t - 24 = 0$ $\quad t = 3.75$ s or -1.30 s

The ball was thrown at $t = 0$, so it hits the ground at a later time, at $t = 3.75$ s.

The ball dropped from rest will require time t_2 to reach the ground, where

$$h = (1/2)gt^2 \quad t = \pm\sqrt{2h/g} = 2.21$$

Thus the second ball should be dropped a time Δt later, where $\Delta t = 3.75$ s – 2.21 s = 1.54 s

EXAMPLE 3.6 A football is kicked at an angle $\theta = 37.0°$ with a velocity of 20.0 m/s. Calculate (a) the maximum height, (b) the time of travel before the football hits the ground, (c) how far away it hits the ground, (d) the velocity vector at the maximum height, and (e) the acceleration vector at maximum height.

Solution

Assume the ball leaves the foot at ground level, and ignore air resistance and rotation of the ball. We decompose the initial velocity into its components

$$v_{0x} = v_0 \cos 37.0° = (20.0\text{m/s})(0.799) = 15.98 \text{ m/s}$$

$$v_{0y} = v_0 \sin 37.0° = (20.0 \text{ m/s})(0.602) = 12.04 \text{ m/s.}$$

(a) We consider a time interval that begins just after the football loses contact with the foot until it reaches its maximum height. During this time interval, the acceleration is g downward. At the maximum height, the velocity is horizontal, so $v_y = 0$: and this occurs at a time given by $\quad v_y = v_{y0} - gt \quad$ with $\quad v_y = 0$. $\quad$ So we get

$t = v_{y0}/g \approx 1.227$ s.

Using the calculated time we get maximum height

$y = v_{0y}t - 1/2gt^2 = 7.35$m.

(b) To find the time it takes for the ball to return to the ground, we consider a different time interval, starting at the moment the ball leaves the foot ($t = 0$, $y_0 = 0$) and ending just before the ball touches the ground ($y = 0$ again). In following equation we set y_0=0 and also $y = 0$ (ground level):

$y = y_0 + v_{y0}\, t - \frac{1}{2}\, g\, t^2$

$0 = 0 + 12.0\, t - \frac{1}{2}\, 9.8\, t^2$

There are two solutions, $t = 0$ (which corresponds to the initial point, y_0), and

$$t = \frac{2 \cdot 12\ \mathrm{m \cdot s^{-1}}}{9.8\ \mathrm{m \cdot s^{-2}}} \approx 2.45\ \mathrm{s}$$

which is the total travel time of the football.

We can see that the time for the whole trip is double the time to reach the highest point. It means the time upwards is the same as the time downwards but only without the air resistance.

(c) The total distance traveled in the x direction equals:

$x = v_{0x}t\ = (16.0\text{m/s})\ (2.45\text{s}) = 39.2\ \text{m}$.

(d) At the highest point, there is no vertical component to the velocity. There is only the horizontal component (which remains constant throughout the flight), so $v = v_{0x} = v_0 \cos 37.0° = 16.0\ \text{m/s}$.

(e) The acceleration vector is the same at the highest point as it is throughout the flight, which is $9.80\ \text{m/s}^2$ downward. In this example we consider the football as if it were a particle, ignoring its rotation. We also neglected air resistance, which is considerable on a rotating football, so our results are not very accurate.

EXAMPLE 3.7 Imagine we are sitting upright in a cabriolet which is moving at constant speed. Then we throw a ball straight upward (from our own point of view), while the car continues to travel forward at constant speed. If air resistance is neglected, will the ball land

(a) behind the car, (b) in the car, or (c) in front of the car?

Solution

We throw the ball straight up from our own reference frame with initial velocity v_{y0}. But when viewed by someone on the ground, the ball also has an initial horizontal component of velocity equal to the speed of the car v_{x0}. Thus, to a person on the ground, the ball will follow the path of a projectile. The ball experiences no horizontal acceleration, so v_{x0} will stay constant and equal to the speed of the car. As the ball follows its arc, the car will be directly under the ball all the time because they have the same horizontal velocity. When the ball falls down, it will drop right into our hands.

EXAMPLE 3.8 A military jet fighter plane flying at 180 m/s pulls out of a vertical dive by turning upward along a circular path of radius 860 m. What is the acceleration of the plane? Express the acceleration as a multiple of g.

Solution

$a = \frac{v^2}{r} = \frac{(180 m/s)^2}{860 m} = 37.7\ \text{m/s}^2 = 37.7\ \text{m/s}^2 / g\ \text{m/s}^2 = 3.8\ g$

EXAMPLE 3.9 A ball tied to the end of a string 0.50 m in length rotates in a vertical circle under the influence of gravity. When the string makes an angle θ = 20° with the vertical, the ball has a speed of 1.5 m/s.

(a) Find the magnitude of the radial component of acceleration at this instant.

Solution

$$a_{\mathrm{r}} = \frac{v^2}{r} = \frac{(1.5\ \mathrm{m/s})^2}{0.5\ \mathrm{m}} = 4.5\ \mathrm{ms}^{-2}$$

(b) What is the magnitude of the tangential acceleration when θ = 20°?

Solution

When the ball is at an angle θ to the vertical, it has a tangential acceleration of magnitude $g \sin\theta$ (the component of g tangent to the circle). Therefore, at θ = 20°,

$$a_{\mathrm{t}} = g \sin 20° = 3.4\ \mathrm{ms}^{-2}$$

(c) Find the magnitude and direction of the total acceleration at θ = 20°.

Solution

Because $\boldsymbol{a} = \boldsymbol{a_r} + \boldsymbol{a_t}$, the magnitude of $\boldsymbol{a}$ at θ =20° is

$$a = \sqrt{{a_{\mathrm{r}}}^2 + {a_{\mathrm{t}}}^2} = \sqrt{4.5^2 + 3.4^2} = 5.6\ \mathrm{ms}^{-2}$$

If ∅ is the angle between $\boldsymbol{a}$ and the string, then

$$\emptyset = \tan^{-1}\left(\frac{a_{\mathrm{t}}}{a_{\mathrm{r}}}\right) = \tan^{-1}\left(\frac{3.4}{4.5}\right) = 37°$$

Note that $\boldsymbol{a}$, $\boldsymbol{a}_{\mathrm{t}}$, and $\boldsymbol{a}_{\mathrm{r}}$ all change in direction and magnitude as the ball rotates through the circle. When the ball is at its lowest elevation (θ = 0), $\boldsymbol{a}_{\mathrm{t}}$ = 0 because there is no tangential component of $\boldsymbol{g}$ at this angle; also, $\boldsymbol{a}_{\mathrm{r}}$ is a maximum because $\boldsymbol{v}$ is a maximum. If the ball has enough speed to reach its highest position (θ = 180°), then $\boldsymbol{a}_{\mathrm{t}}$ is again zero but $\boldsymbol{a}_{\mathrm{r}}$ is a minimum because $\boldsymbol{v}$ is now a minimum. Finally, in the two horizontal positions (θ = 90° and 270°), $\boldsymbol{a_t} = \boldsymbol{g}$ and $\boldsymbol{a}_{\mathrm{r}}$ has a value between its minimum and maximum values.

EXAMPLE 3.10

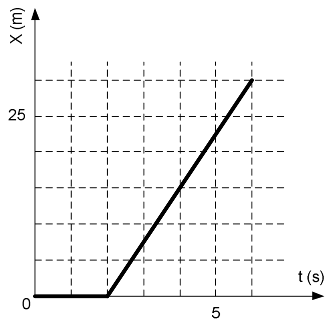


Fig. 3.5 Graph: distance as a function of time

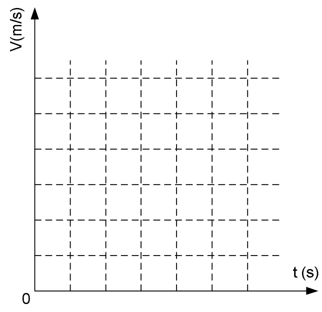


Fig. 3.6 Graph: velocity as a function of time (to be done by the reader)

Describe in words the motion of the object from 0 to 6.0 s.

What is the instantaneous velocity of the object at the following times: t = 1 s, t = 3 s?

What is the simple average of these two velocities?

What is the velocity for the entire interval?

Why are these two values different? Which is correct?

Graphically represent the relationship between velocity and time for the object described above.

From your velocity versus time graph determine the total displacement of the object.

EXAMPLE 3.11

The graph below represents the motion of a moving object.

Where on the graph above is the object moving most slowly? (How do you know?)

Where on the graph above is the object speeding up? (How do you know?)

Where on the graph above is the object slowing down? (How do you know?)

Where on the graph above is the object changing direction? (How do you know?)

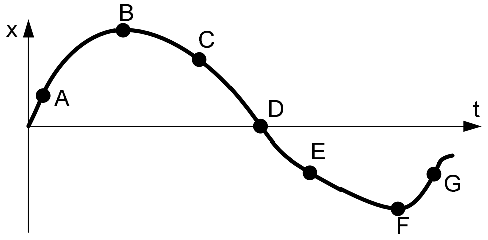
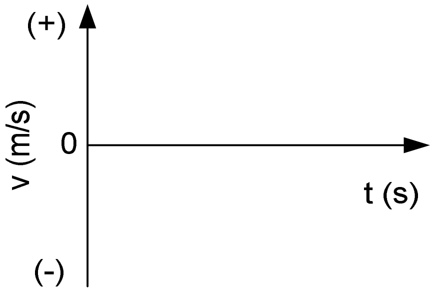
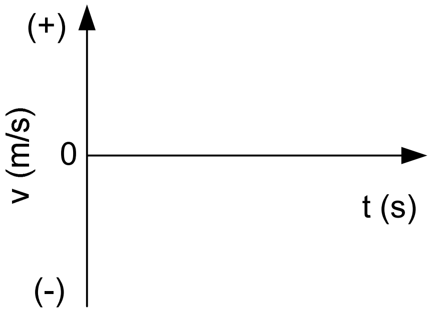
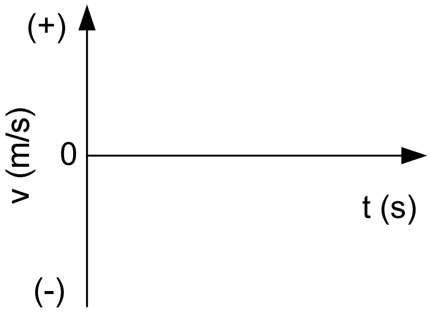
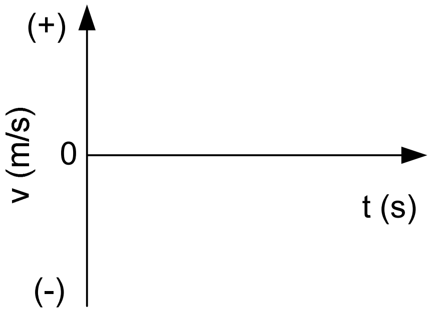
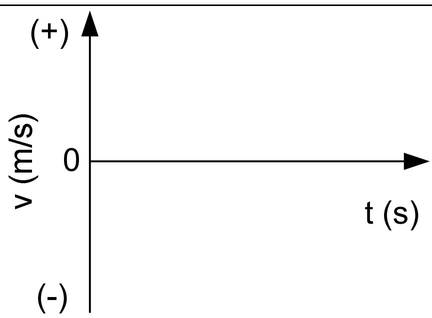


Fig. 3.7 Graph representing path of the motion

EXAMPLE 3.12 Worksheet – free particle model – constant acceleration

The problem	v vs t graph	Solution
1. A poorly tuned Yugo can accelerate from rest to a speed of 28 m/s in 20 s. a) What is the average acceleration of the car? b) What distance does it travel in this time?		
2. At t = 0 a car has aspeed of 30 m/s. After 6 s, its speed is 15 m/s. What is its average acceleration during this time interval?		
3. A bear spies some Honey and takes off from rest, accelerating at a rate of 2.0 m/s^2. If the honey is 10 m away, how fast will his snout be going at the moment of ecstasy?		
4. A bus moving at 20 m/s (t = 0) slows at a rate of 4 m/s each second. a) How long does it take the bus to stop? b) How far does it travel while braking?		
5. A car whose initial speed is 30 m/s slows uniformly to 10 m/s in 5 seconds. a) Determine the acceleration of the car. b) Determine the distance it travels in the 3rd second.		

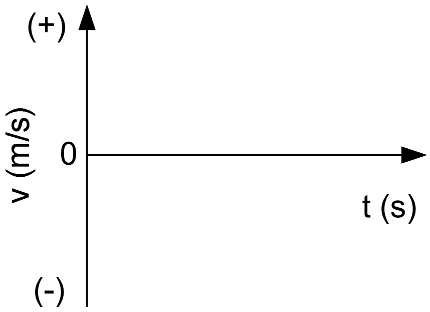
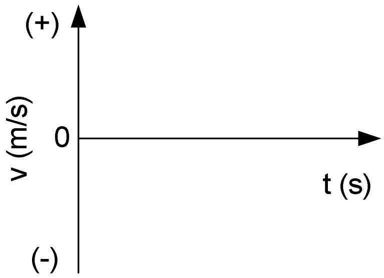
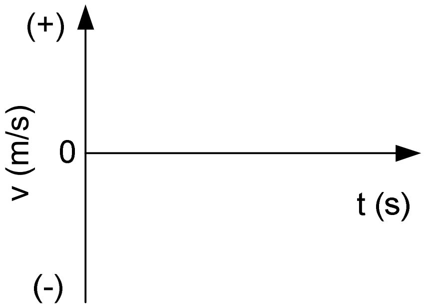
6. A dog runs down his driveway with an initial speed of 5 m/s-for 8 s, then uniformly increases his speed to 10 m/s in 5 s. a) What was his acceleration during the 2nd part of the motion? B) How long is the driveway?		
7. A physics student skies down a slope accelerating at a constant 2.0 m/s^2. If it takes her 15 s to reach the bottom, what is the length of the slope		
8. A mountain goat starts a rock-slide and the rocks crash down the slope 100 m. If the rocks reach the bottom in 5 s, what is its acceleration?		

Fig. 3.8 Free particle worksheet

EXAMPLE 3.13 The track of a cosmic ray particle in a photographic emulsion is found empirically to be described by the expression $\boldsymbol{r} = (3t^2 - 6t)\boldsymbol{i} + (5 - 8t^4)\boldsymbol{j}$. Determine the velocity and acceleration. ($\boldsymbol{v} = (6t - 6)\,\boldsymbol{i} + (-32t^3)\,\boldsymbol{j}$, $\boldsymbol{a} = 6\boldsymbol{i} + (-96t^2)\,\boldsymbol{j}$)

EXAMPLE 3.14 A motorist drives 120 km at 100 km/h and 20 km at 30 km/h. What is his average speed for the trip? (75 km/h)

EXAMPLE 3.15 A motorist drives half an hour at 100 km/h and two hours at 80 km/h. What is his average speed for the trip? (84 km/h)

EXAMPLE 3.16 A cheetah is the fastest land mammal and it can run at speeds of about 101 km/h for a period of perhaps 20 s. The next fastest land animal is an antelope, which can run at about 88 km/h for a much longer time. Suppose a cheetah is chasing an antelope, and both are running at top speed, (a) If the antelope has a 40 m head start, how long will it take the cheetah to catch him, and how far will the cheetah travel in this time? (b) What is the maximum head start the antelope can have if the cheetah is to catch him within 20 s (at which time the cheetah runs out of breath)? ((a) 6.9 s, 311.2 m, (b) 116 m)

EXAMPLE 3.17 A typical jet fighter plane launched from an aircraft carrier reaches a take-off speed of 280 km/h in a launch distance of 95 m. (a) Assuming constant acceleration, calculate the acceleration in meters per second, (b) How long does it take to launch the fighter? ((a) 32.3 m/s^2), (b) 2.4 s)

EXAMPLE 3.18 A motorist traveling 31 m/s passes a stationary motorcycle police officer. 2.5 s after the motorist passes, the police officer starts to move and accelerates in pursuit of the speeding motorist. The motorcycle has constant acceleration of 3.6 m/s^2. (a) How fast will the police officer be traveling when he overtakes the car? Draw curves of x versus t for both the motorcycle and the car, taking t = 0 at the moment the car passes the stationary police officer, (b) Suppose that for reasons of safety the policeman does not exceed a maximum speed of 45 m/s. How long will it then take him to overtake the car, and how far will he have traveled? ((a) 83 m/s, (b) 25.6 s, 872 m)

EXAMPLE 3.19 Suppose that motion studies of a runner show that the maximum speed he can maintain for a period of about 10 s is 12 m/s. If in a 100-m dash this runner accelerates with constant acceleration until he reaches this maximum speed and then maintains this speed for the rest of the race, what acceleration will he require if his total time is 11 s? (2.25 m/s^2)

EXAMPLE 3.20 A typical jet liner lands at a speed of 100 m/s. While braking, it has an acceleration of -5.2 m/s^2. (a) How long does it take to come to a stop? (b) What is the minimum length of the landing strip under these conditions? ((a) 19.2 s, (b) 962 m)

EXAMPLE 3.21 On a 40 km bike ride a cyclist rides the first 20 km at 20 km/h. What speed is required for the final 20 km if the average speed for the trip is to be (a) 10 km/h? (b) 30 km/h? ((a) 6.7 km/h, (b) 60 km/h)

4 DYNAMICS

Dynamics studies how forces influence motion. Dynamics is involved in understanding a wide range of phenomena. The basic ideas needed are stated as Newton's laws of motion.

Newton's first law: If the net external force F = 0, then a = 0 and v = constant

The tendency of an object to maintain its state of rest or of uniform motion in a straight line is called inertia. As a result, Newton's first law is often called the law of inertia. Newton's first law makes no distinction between an object at rest and an object moving with constant velocity. Whether an object is at rest or is moving with constant velocity depends on the reference frame in which the object is observed. A reference frame in which the law of inertia holds exactly is called an inertial reference frame and any reference frame moving with constant velocity relative to an inertial reference frame is also an inertial reference frame.

Newton's second law: $\boldsymbol{F} = m\boldsymbol{a}$

Newton's third law: If A exerts force $\boldsymbol{F}_{AB}$ on B and B exerts force $\boldsymbol{F}_{BA}$ on A, then $\boldsymbol{F}_{AB} = -\boldsymbol{F}_{BA}$

Equilibrium: If $\boldsymbol{F} = 0$, then $\boldsymbol{a} = 0$, and we can write $\boldsymbol{F}_x = 0$, $\boldsymbol{F}_y = 0$, $\boldsymbol{F}_z = 0$

EXAMPLE 4.1 Calculate the sum of two force vectors acting on the small boat in Fig. 4.1.

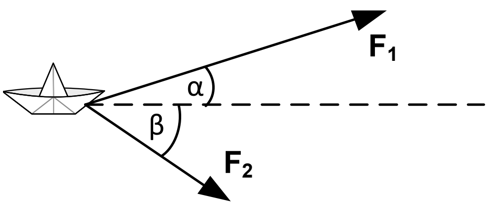


Fig. 4.1 The sum of two force vectors

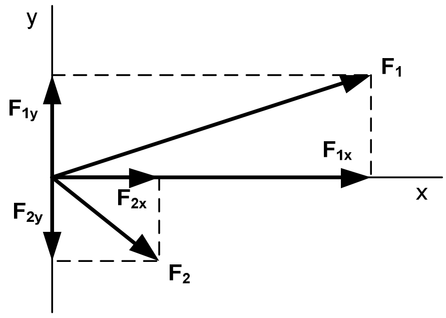


Fig. 4.2 Decomposing of vectors

Solution

After decomposing these two vectors (Fig. 4.2) we can express the components of $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$, as follows:

$$F_{1x} = F_1 \cos\alpha \quad F_{1y} = F_1 \sin\alpha \qquad F_{2x} = F_2 \cos\beta \quad F_{2y} = F_2 \sin\beta$$

where F_1, F_2 are the magnitudes of the vectors $\boldsymbol{F}_1$, $\boldsymbol{F}_2$, respectively. As the angle β has a negative sign, the component F_{2y} is negative and it points along the negative y axis. Components of the resultant force are given as the sum of the components of the forces $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$

$F_{Rx} = F_{1x} + F_{2x}$, $\quad F_{Ry} = F_{1y} + F_{2y}$

To find the magnitude of the resultant force, we use the Pythagorean theorem, so

$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$

and for the angle φ which the force F_R makes with the x axis we use $\tan\varphi = \frac{F_{Ry}}{F_{Rx}}$

EXAMPLE 4.2 Calculate the constant force acting horizontally required to accelerate the 20 kg box of chocolate from rest to 0.5 m/s in 2 s (friction is negligible).

Solution

There are three forces acting on the box. The forward pushing force $\boldsymbol{F}_\mathbf{p}$ exerted by the person, the downward force of gravity $\boldsymbol{F}_\mathbf{g}$ and the upward force $\boldsymbol{F}_\mathbf{N}$ exerted by the floor (which is the reaction to the force of the cart pushing down on the floor). The sum of both vertical forces $\boldsymbol{F}_\mathbf{g}$ and $\boldsymbol{F}_\mathbf{N}$ must be zero; if it did not the box would accelerate vertically.

So $\quad F_N = F_g = m\,g = 196$ N

Then the net force on the box is simply $\boldsymbol{F}_\mathbf{p}$

To calculate how large $\boldsymbol{F}_\mathbf{p}$ must be, we first calculate the acceleration required:

$a = \frac{v}{t} = \frac{0.5}{2} = 0.25 \text{ m/s}^2$.

So, the magnitude of the force exerted by the person must be $F_p = ma = 20 \cdot 0.25 = 5\,\text{N}$

EXAMPLE 4.3 A box of chocolate mass m is being pulled by a person along the surface of a table with a force $\boldsymbol{F}_\mathbf{p}$. The force is applied at an angle α. The friction is assumed to be neglected. Calculate

a) the acceleration of the box,

b) the magnitude of the upward force $\boldsymbol{F}_N$ exerted by the table on the box.

Solution

We decompose all forces into components

$\boldsymbol{F}_\mathbf{p} = (F_p \cos\alpha, F_p \sin\alpha)$ $\qquad \boldsymbol{F}_\mathbf{g} = (0, -mg)$ $\qquad \boldsymbol{F}_\mathbf{N} = (0, F_N)$

In the horizontal x direction, $\boldsymbol{F}_\mathbf{N}$ and $\boldsymbol{F}_\mathbf{g}$ have zero components, thus $F_{px} = m\,a_x$. So:

$$a_x = \frac{F_{px}}{m} = \frac{F_p \cos\alpha}{m}$$

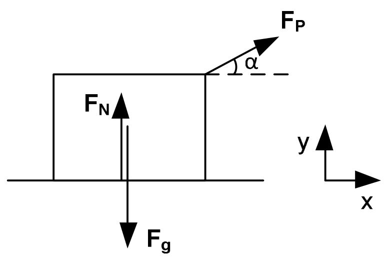


Fig.4.3 Free body diagram

In the vertical direction we have: $\quad m\,a_y = F_{Ny} + F_{py} + F_{gy}$.

We know $a_y = 0$ since the box does not move vertically. Then:

$(F_{Ny} = F_N,\ F_{gy} = -mg,\ F_{py} = F_p \sin\alpha)$ $\qquad 0 = F_N + F_p \sin\alpha - mg$

And $\qquad F_N = mg - F_p \sin\alpha$

Notice that F_N is less than F_g. The ground does not push against the full weight of the box since part of the pull force exerted by the person is in the upward direction.

EXAMPLE 4.4 Two boxes connected by a lightweight cord are resting on a table. The boxes have masses m_1 and m_2. A horizontal force of $\boldsymbol{F}_\mathrm{p}$ is applied to the right box as shown in Fig.4.4 (friction is neglected). Find:

a) the acceleration of boxes,

b) the tension $\boldsymbol{T}$ in the cord.

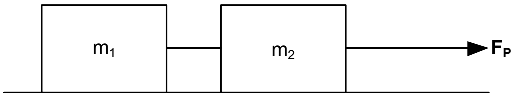


Fig.4.4 Force applied to two boxes

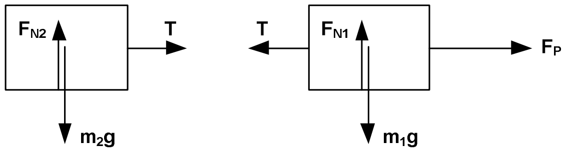


Fig. 4.5 Free body diagram

Solution

We draw the force diagram for each of the boxes. We can neglect the cord mass relative to the mass of the boxes. The force $\boldsymbol{F}_\mathrm{p}$ acts on the box m_1; box m_1 exerts a force $\boldsymbol{T}$ on the connecting cord and the cord exerts a force $-\boldsymbol{T}$ back on box m_1 (the third law of motion). Because the cord is considered to be massless, the tension at each end is the same. So the cord exerts a force $\boldsymbol{T}$ on the box m_2. The acceleration of both boxes is the same. For the horizontal motion we have:

For box m_1: $\quad m_1 a = F_\mathrm{p} - T$, $\qquad$ for box m_2: $\quad m_2 a = T$

Hence $\qquad m_2 a = F_\mathrm{p} - m_1 a \qquad a = \frac{F_p}{m_1 + m_2}$

For the tension T we have $\quad T = m_2 a$, $\qquad$ or $\qquad T = F_\mathrm{p} - m_1 a$.

EXAMPLE 4.5 Suppose the cord in previous problem is a heavy rope of mass m. Calculate the acceleration of each box and the tension in the rope.

Solution

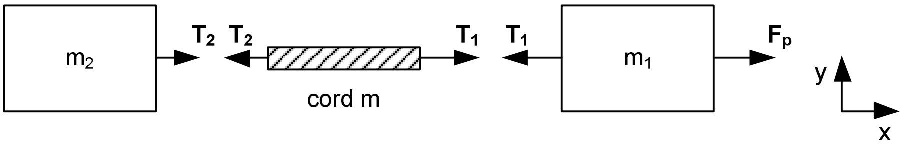


Fig.4.6 Free body diagram

Since the cord has mass m, the product ma will not be zero, so the forces (tensions) at either end will not be the same. Therefore $T_1 - T_2 = m\,a$.

For box m_1 we have: $F_p - T_1 = m_1\,a$, for box m_2: $T_2 = m_2\,a$

We now have obtained three unknowns T_1, T_2 and a in three equations. The sum of all three equations yields: $a = F_p/(m + m_1 + m_2)$.

Then $T_2 = m_2 a = F_p \frac{m_2}{m+m_1+m_2}$ and $T_1 = F_p - m_1 a = F_p \frac{m+m_2}{m+m_1+m_2}$

EXAMPLE 4.6 Suppose two different boxes ($m_2 > m_1$) are placed with the cord joining them hanging over a frictionless massless pulley as in figure bellow. We assume the cord is massless. Calculate the acceleration of boxes and the tension in the cord.

Solution

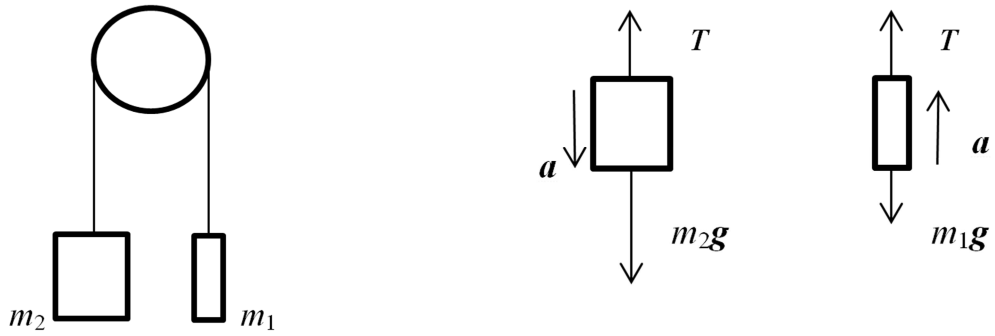


Fig. 4.7 Free body diagram

Since the cord is massless the tension T is the same at both ends. As m_2 is heavier, it accelerates downward and box m_1 accelerates upward.

To find its value we write the second law of motion for each box, taking the upward direction as positive $T - m_1\,g = m_1\,a$, $T - m_2\,g = -m_2\,a$

Solving these two equations we get $a = \frac{m_2 - m_1}{m_2 + m_1} g$

The tension T we can get from either of the two equations above

$T = (g + a)\,m_1$ $T = (g - a)\,m_2$

EXAMPLE 4.7 Here is a famous classic problem that will make you think. A rope is passed over a pulley suspended from a tree branch, and a stalk of bananas is tied to one end. A monkey hangs from the other end of the rope, and the mass of the bananas and the monkey are balanced. Now the monkey starts climbing up the rope. What will happen to the bananas? Will they stay in the same place, or will they move up away from the ground, or will they move down toward the ground?

Solution

Look at the force diagram for the monkey. His weight $\boldsymbol{mg}$ acts downward, and the rope tension $\boldsymbol{T}$ acts upward. If the monkey is to start moving up from rest, he must accelerate upward, which means there must be net upward force acting on him. The net upward force on the monkey is $T - mg$. But the tension is the same everywhere in a rope, so the tension at the end of the rope attached to the bananas is also T, greater than mg. Thus the bananas experience the same upward force as does the monkey, and so the bananas will move up with the same acceleration and velocity as the monkey. Both will move higher from the ground at the same rate. The net upward force on the system made up of the bananas plus the monkey is provided by the pulley.

EXAMPLE 4.8 A person whose weight is 600 N stands on a bathroom scale in an elevator. What will the scale read when the elevator is (a) moving up or down at constant speed? (b) Accelerating up with acceleration 0.5 $\boldsymbol{g}$? (c) Accelerating downward with acceleration 0.5 $\boldsymbol{g}$? (d) Accelerating downward with acceleration $\boldsymbol{g}$?

Solution

The force diagram includes two forces: the gravity force $G = mg = 600$ N downward and the normal force $\boldsymbol{N}$, exerted upward by the surface of the scale. This normal force is the scale reading.

(a) Constant velocity means $a = 0$, so equilibrium and $N = G$

(b) $N - G = ma = +0.5\ mg = 0.5\ G$, so $N = G + 0.5\ G = 1.5\ G$

(c) $N - G = ma = -0.5\ mg = -0.5\ G$, so $N = \mathrm{G} - 0.5\ G = 0.5\ G$

(d) $N - G = ma = -mg = -G$, so $N = 0$

The last case, when the scale reading is zero, represents what is called "effective weightlessness". The elevator is falling with acceleration $-g$, as is the person. Thus the person does not press down on the elevator. He is seemingly "weightless". This is the situation with the astronauts in an orbiting space vehicle. The vehicle and everything in it are falling freely, and hence they all seem weightless. You have probably seen pictures where the astronauts, their pencils, and their sandwiches and other loose equipment float weightlessly around the spaceship

EXAMPLE 4.9 A small object of mass m is suspended from a string of length L. The object revolves with constant speed v in a horizontal circle of radius r (because the string sweeps out due surface of a cone, the system is known as a conical pendulum). Find an expression for v.

Solution

Fig. 4.8 Conical pendulum

Let θ be the angle between string and vertical. The force $\boldsymbol{T}$ exerted by the string is resolved into a vertical component $T \cdot \cos\theta$ and a horizontal component $T \cdot \sin\theta$. Since the object does not accelerate in the vertical direction, $\sum F_y = ma_y = 0$, and the upward vertical component of $\boldsymbol{T}$ must balance the downward force of gravity. So,

(1) $T \cdot \cos\theta = mg$.

Since the radial acceleration is $T \cdot \sin\theta$, the Newton's second law yields:

(2) $\sum F_r = T \cdot \sin\theta = ma_r = mv^2/r$.

Dividing (2) by (1) we eliminate T:

$\tan\theta = v^2/(rg)$ or $v = \sqrt{rg\tan\theta}$

Since $r = L \cdot \sin\theta$,

$$v = \sqrt{Lg\sin\theta\tan\theta}.$$

Note, the speed is independent of the mass of the object!

EXAMPLE 4.10 A girl moves her brother on a sled at a constant velocity by exerting a force $\boldsymbol{F}$. The coefficient of friction between the sled and the ground is 0.05. The sled and rider have a mass of 20 kg. What force is required if (a) she pushes on the sled at an angle of 30° below horizontal? (b) She pulls the sled at an angle of 30° above horizontal?

Solution

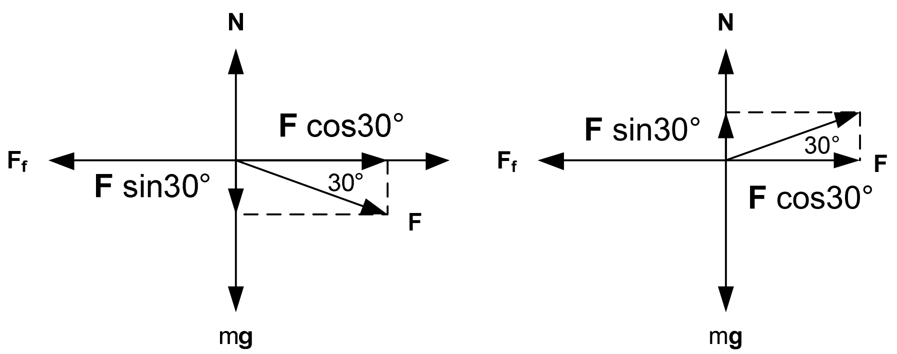


Fig. 4.9 Free body diagram: a) pushing b) pulling

a) $N = m\,g + F\sin 30°$ $F_f = \mu N = F\cos 30°$ so

$\mu\,(mg + F \sin 30°) = F \cos 30°$

$$F = \frac{\mu\, m\, g}{\cos 30° - \mu \sin 30°} = \frac{0.05 \cdot 20\,\mathrm{kg} \cdot 9.8\,\mathrm{m} \cdot \mathrm{s}^{-2}}{\cos 30° - 0.05 \cdot \sin 30°} = 11.7N$$

b) $N + F \sin 30° = m\, g$ $\quad F_f = \mu N = F \cos 30°$ $\quad$ so

$\mu\,(mg - F \sin 30°) = F \cos 30°$

$$F = \frac{\mu\, m\, g}{\cos 30° + \mu \sin 30°} = \frac{0.05 \cdot 20kg \cdot 9.8 m \cdot s^{-2}}{\cos 30° + 0.05 \cdot \sin 30°} = 11.0N$$

The force required in (b) is less than for (a) because in (b) the force ***F*** angles up and supports some of the weight. This reduces ***N*** and hence $\boldsymbol{F_f}$.

EXAMPLE 4.11 A coin is placed on a turntable turning at 100/3 RPM (revolutions per minute). What is the coefficient of friction between the coin and the turntable if the maximum radius, before the coin slips, is 0.14 m?

Solution

The frictional force F_f between the coin and turntable provides the center-directed force to keep the coin on the turntable. This center-directed force must equal mv^2 / r.

$F_f = \mu N = \mu\, m\, g$ $\qquad \mu\, m\, g = mv^2 / r$

or $\qquad \mu = v^2 / (r\, g)$

$$v = 2\pi r\ f = 2\pi .0.14 m \frac{100}{3} \frac{1}{\min} \frac{1\min}{60 s} = 0.49 m \cdot s^{-1}$$

Therefore, $\mu = 0.49^2 / (0.14 \cdot 9.8) = 0.175$

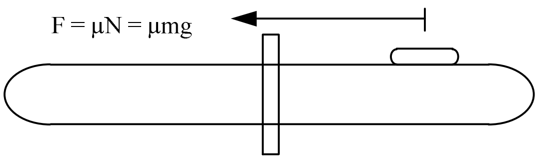


Fig.4.10 Free body diagram

EXAMPLE 4.12 The banked exit ramp. Curved exit ramp for a highway is tilted toward the inside of the curve, so a car moving at the designated speed can negotiate the curve even when the road is covered with ice. Let the designated speed be 13.4 m/s and the radius is 50 m. At what angle should the curve be banked?

Solution

On an untilted road, the force that causes the radial acceleration is the force of static friction between car and road. However, if the road is tilted at an angle θ, the normal force ***N*** has a horizontal component $N{\cdot}\sin\theta$ pointing to the center of the curve. Because the ramp is to be designed so that the force of static friction is zero, only the component $N{\cdot}\sin\theta$ causes the radial acceleration. So, the Newton`s second law gives for the radial direction:

$\sum F_r = N \sin\theta = mv^2/r$

The car is in equilibrium in the vertical direction. Thus, form $\sum F_y = 0$, we have

$N\cos\theta = mg$

Dividing (1) by (2) gives $\tan\theta = v^2/(rg)$.

So

$$\theta = \tan^{-1}[v^2/(rg)] = 20.1°$$

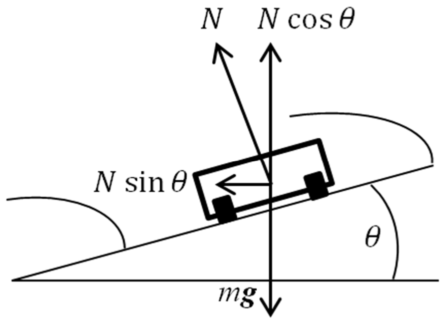


Fig.4.11 Car on the banked curve

If a car rounds the curve at a speed less (more) than 13.4 m/s, friction is needed to keep it from sliding down (up) the bank. The banking angle is independent of the mass of the vehicle.

EXAMPLE 4.13 An object falling through the air at high speed experiences a drag force $\boldsymbol{F}_D$ which can be expressed approximately as $F_D = C_D \rho_A A v^2/2$

Here ρ_A is the density of air, C_D is a drag coefficient that depends on the shape and texture of the falling object, and A is the projected area of the object as seen looking up from the ground. C_D is a dimensionless number between 0 and 1.

(a) Determine the maximum speed (called the terminal velocity v_T) a falling object reaches in the presence of this drag force,

(b) How does the terminal velocity of an object depend on its size? To answer this, calculate the ratio of the terminal velocities for two spherical hailstones, one of radius R_1 and a larger one of radius R_2

Solution

(a) As the object falls, v gets larger and larger, until finally v gains the terminal velocity v_T

$$F = \frac{\rho_A A C_D v^2}{2} - mg = ma$$

When v_T = constant, a = 0, thus $v_T = \sqrt{\dfrac{2mg}{C_D \rho_A A}}$

(b) For a sphere of ice, m = (density).(volume), $m = \rho 4\pi R^3/3$. The projected area seen from below is $A = \pi R^2$. Thus

$$v_T = \sqrt{\frac{2mg}{C_D\rho_A A}} = \sqrt{\frac{2(\rho 4\pi R^3/3)g}{C_D\rho_A(\pi R^2)}} = \sqrt{\frac{8\rho g}{3C_D\rho_A}}\sqrt{R} = \kappa\sqrt{R}\,, \qquad \text{where} \quad \kappa = \sqrt{\frac{8\rho g}{3C_D\rho_A}}$$

For two ice spheres, $\frac{v_{T2}}{v_{T1}} = \frac{\kappa\sqrt{R_2}}{\kappa\sqrt{R_1}} = \sqrt{\frac{R_2}{R_1}}$

Thus large objects fall faster than small ones of the same shape. Comparing the kinetic energies *KE* of these two objects yields:

$$\frac{KE_2}{KE_1} = \frac{mv_{\mathrm{T}2}^2/2}{mv_{\mathrm{T}1}^2/2} = \frac{\rho(4/3)\pi R_2^3 v_{\mathrm{T}2}^2}{\rho(4/3)\pi R_1^3 v_{\mathrm{T}1}^2} = \frac{R_2^3 v_{\mathrm{T}2}^2}{R_1^3 v_{\mathrm{T}1}^2} = \frac{R_2^3}{R_1^3}\left(\frac{v_{\mathrm{T}2}}{v_{\mathrm{T}1}}\right)^2 = \frac{R_2^3}{R_1^3}\frac{R_2}{R_1} = \left(\frac{R_2}{R_1}\right)^4 = \left(\frac{D_2}{D_1}\right)^4$$

So, a hail having diameter D_2 = 6 mm has kinetic energy $(6/2)^4 = 81$-times bigger than a hail with diameter D_1 = 2 mm. Big hailstones can flatten a wheat field, whereas small ones do not hurt it.

EXAMPLE 4.14 A manufacturer quotes an aerodynamic constant C = 0.30 for an automobile of mass 900 kg and cross-sectional area A = 2.8 m^2. If the driver were to coast (in neutral) down a long hill with a slope of 8.0° (see Fig.4.12), what would be the terminal velocity? Recall that terminal velocity is the constant final velocity of a body moving under the combined influence of gravity and air resistance. Assume that air resistance is the only source of friction.

Solution

We need only consider the components of the forces parallel to the motion.

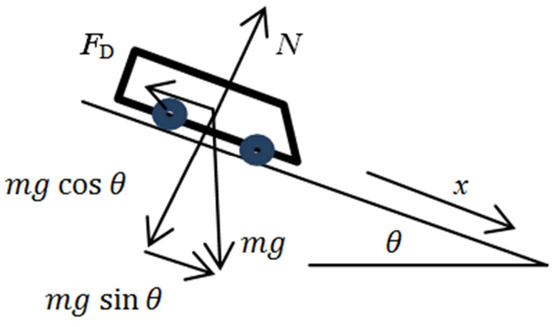


Fig.4.12 Free body diagram

For constant terminal velocity v_T, $\boldsymbol{F}_D$ balances component of weight along the slope.

$G_x = mg\sin\theta$ and

$F_{D,x} = C_D\rho_A A v^2/2$

When terminal (constant) velocity is reached, the two forces balance and the acceleration is zero. Thus, $C_D\rho_A A v_T^2/2 = mg\sin\theta$

Solving for the velocity, we find $v_{\mathrm{T}} = \sqrt{\frac{2mg\sin\theta}{C_{\mathrm{D}}\rho_{\mathrm{A}}A}}$

Inserting the values given in the problem, plus the air density ρ_A = 1.3 kg/m^3

$$v_T = \sqrt{\frac{2(900\ \mathrm{kg})(9.81\ \mathrm{m \cdot s^{-2}}) \sin 8°}{0.3(1.3\ \mathrm{kg \cdot m^{-3}})(2.8\ \mathrm{m^2})}} = 47\ \mathrm{m/s}$$

On such a slope, considerable speed can be attained before air resistance limits the motion.

EXAMPLE 4.15 The oscillations of simple pendulum can be analyzed in terms of force. For small oscillations the tension in the cable can be written in components: one vertical and the other - the restoring force, the force that returns the mass to equilibrium.

Solution

For small angle oscillations: $F\cos\theta \cong F$ and $F\sin\theta \cong -\frac{mgx}{l} = -(mg/l)x$, with x being the displacement, approximately equal to the arc length. As the acceleration is caused by the *x*-component of *F*, the Newton`s second law yields*:*

$$m\frac{\mathrm{d}^2}{\mathrm{d}t^2}x = -\frac{mg}{l}x. \qquad \text{As} \quad x \cong l\theta:$$

$$m\frac{\mathrm{d}^2}{\mathrm{d}t^2}l\theta = -\frac{mg}{l}l\theta \qquad \text{or} \qquad \frac{\mathrm{d}^2}{\mathrm{d}t^2}\theta = -\frac{g}{l}\theta$$

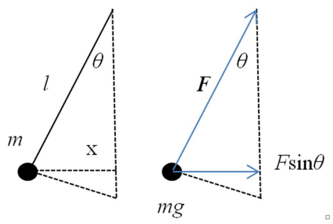


Fig.4.13 Simple pendulum

As we know, the solution of this equation is: $\theta = \theta_0 \cos(\omega t + \varphi)$.

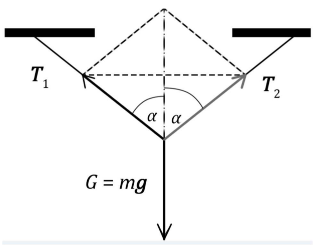
Then putting the second derivation of θ into the last equation we get the results:

$$\omega = \sqrt{\frac{g}{l}} \qquad \text{and} \qquad T = 2\pi\sqrt{\frac{l}{g}}$$

The most important point to note is that the period *T* is independent of the mass. Historically this was one of the first instruments to measure "*g*" - the gravitational constant.

EXAMPLE 4.16 A 1 500-kg car moving on a flat, horizontal road negotiates a curve. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.500, find the maximum speed the car can have and still make the turn successfully. (13.1 m/s)

EXAMPLE 4.17 A body of mass *m* hangs from two cables fastened symmetric to a support as in Figure 4.14. Calculate the magnitude *T* of tensions $\boldsymbol{T_1},\ \boldsymbol{T_2}$ in the cables.



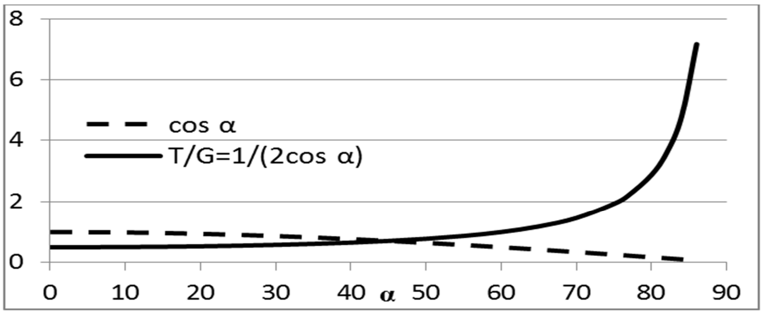


Fig.4.14 Free body diagram *Fig.4.15 The dependence of the ratio T/G on the angle α*

Solution

We decompose the weight $\boldsymbol{G}$ into directions of cables. From the free body diagram we can write: $\cos\alpha = \frac{G/2}{T_1}$ So:

$$T = T_1 = T_2 = \frac{G}{2\cos\alpha} \qquad \text{or} \qquad \frac{T}{G} = \frac{1}{2\cos\alpha}$$

The dependence of the ratio *T/G* on the angle α is shown in Fig.4.15. For small angles the tension is approximately 0.5 *G*. Note the strong increase of the tension for angles bigger than 80°.

Let us next solve this example for an asymmetric case shown in Fig. 4.16.

Solution

We must again decompose the weight $\boldsymbol{G}$ into directions of cables. We know angles and the side *G*, so we can use the sine rule of triangles.

$$\frac{T_1}{\sin\beta} = \frac{G}{\sin(180° - \alpha - \beta)}$$

$$T_1 = G\frac{\sin\beta}{\sin(180° - \alpha - \beta)}$$

$$T_2 = G\frac{\sin\alpha}{\sin(180° - \alpha - \beta)}$$

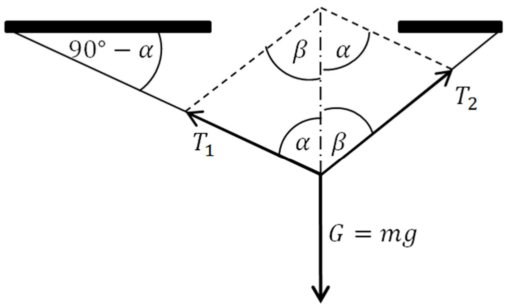


Fig.4.16 Free body diagram

Note, the tensions T_1 and T_2 are proportional to $\sin\beta$ and $\sin\alpha$, respectively. Not vice versa, as one would expect!

Setting $\alpha = \beta$ we should receive the foregoing result - $G/(2\cos\alpha)$. Let us prove it:

$$T_1 = T_2 = \frac{G\sin\alpha}{\sin(180° - 2\alpha)} = \frac{G\sin\alpha}{\sin 2(90° - \alpha)} = \frac{G\sin\alpha}{2\sin(90° - \alpha)\cos(90° - \alpha)}$$
$$= \frac{G\sin\alpha}{2\cos\alpha\sin\alpha} = \frac{G}{2\cos\alpha}$$

EXAMPLE 4.18 A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support as in Figure 4.17. The upper cables make angles of 37.0° and 53.0° with the horizontal. These upper cables are not as strong as the vertical cable and will break if the tension in them exceeds 100 N. Does the traffic light remain hanging in this situation, or will one of the cables break? (T_1 = 73.4 N, T_2 = 97.4 N, both values are less than 100 N, so they will not break)

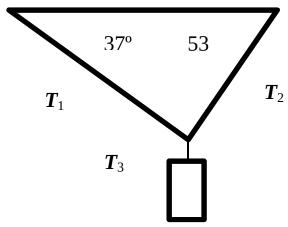


Fig. 4.17 Traffic lights

EXAMPLE 4.19 A car of mass *m* is on an icy driveway inclined at an angle *θ*. Find the acceleration of the car, assuming the driveway is frictionless. So far we were neglecting the drag and friction forces ($a = g\sin\theta$).

EXAMPLE 4.20 A hockey puck on a frozen pond is given an initial speed of 20.0 m/s. If the puck always remains on the ice and slides 115 m before coming to rest, determine the coefficient of kinetic friction between the puck and ice. (μ_k= 0.177)

EXAMPLE 4.21 A body of mass *m* is on an incline at an angle 45˚. After moving 1 m its speed increased from the initial value 1.8 km/h to 12.6 km/h. Determine the coefficient of kinetic friction. (μ_k = 0.135)

EXAMPLE 4.22 A small sphere of mass *m* is attached to the end of a cord of length *L* at an angle *θ* from vertical and set into motion with constant speed ***v*** in a vertical circle about a fixed point *O*. Determine the period *T* of this conical pendulum. ($T = 2\pi\sqrt{\frac{L\cos\theta}{g}}$)

5 WORK, ENERGY, POWER.

A **system** is most often a single particle, a collection of particles or a region of space. A **system boundary** separates the system from the **environment**. Many physics problems can be solved by considering the interaction of a system with its environment.

The **work** *W* done on a system by an agent exerting a constant force ***F*** on the system is the product of the magnitude Δr of the displacement of the point of application of the force and the component F cos Θ of the force along the direction of the displacement Δr:

$$W = \boldsymbol{F} \cdot \Delta \boldsymbol{r} = F\,\Delta r \cos\Theta$$

where the result is a scalar quantity and Θ is the angle between the two vectors. The scalar product obeys the commutative and distributive laws.

If a varying force does work on a particle as the particle moves along the *x* axis form x_i to x_f, the work done by the force on the particle is given by

$$W = \int_{x_i}^{x_i} F_x \mathrm{d}x$$

Where F_x is the component of force in the *x*-direction.

The **kinetic energy** *KE* of a particle of mass *m* moving with a speed *v* is

$$KE = (1/2)mv^2$$

The **work - kinetic energy theorem** states that if work is done on a system by external forces and the only change of the system is in its speed, then

$$\sum W = KE_f - KE_i = (1/2)mv_f^2 - (1/2)mv_i^2$$

For a non-isolated system, we can equate the change in the total energy stored in the system to the sum of all the transfers of energy across the system boundary. For an isolated system, the total energy is constant – this is a statement of **conservation of energy**.

If a kinetic friction force $\boldsymbol{F}_{kf}$ acts along a distance *d*, the kinetic energy of the system is reduced and the appropriate equation to be applied is

$$\Delta KE = KE_f - KE_i = -F_{kf} \cdot d + \sum W_{other\,forces}$$

or

$$KE_f = KE_i - F_{kf} \cdot d + \sum W_{other\,forces}$$

The **instantaneous power** *P* is defined as the time rate of energy transfer. If an agent applies a force ***F*** to an object moving with a velocity ***v***, the power *P* delivered by that agent is

$$P = \frac{\mathrm{d}W}{\mathrm{d}t} = \boldsymbol{F} \cdot \boldsymbol{v}$$

EXAMPLE 5.1 A roller-coaster car of mass m glides down to the bottom of a straight section of inclined track from a height h. (a) What is the work done by gravity on the car? (b) What is the work done by the normal force? Treat the motion as particle motion.

Solution

(a) Figure 5.1 shows the inclined track. The roller-coaster car moves down the full length of this track. By inspection of the right triangle formed by the incline and the ground, we see that the displacement of the car has a magnitude $s = \frac{h}{\sin\theta}$

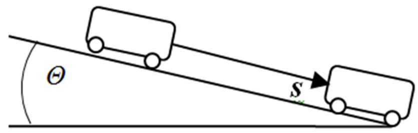


Fig.5.1 A roller- coaster car

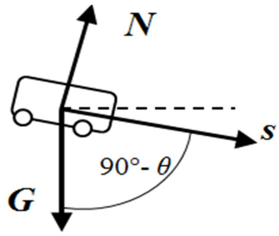


Fig.5.2 Free body diagram

Figure 5.2 shows a "free-body" diagram for the car on an incline of height *h*; the forces acting on it are the normal force ***N*** and the weight ***G***.

The weight ***G*** makes an angle (90° - *θ*) with the displacement ***s***. The work *W* done by the weight ***G*** along the path ***s*** is

$$W = \boldsymbol{G} \cdot \boldsymbol{s} = Gs\cos(90° - \theta) = mg\frac{h}{\sin\Theta}\cos(90° - \theta) = mg\frac{h}{\sin\Theta}\sin\theta = mgh$$

(b)The work done by the normal force is zero, since this force makes an angle of 90º with the displacement. The important thing from this result is that the work done by the weight is independent of the angle of the incline - it depends only on the change of height, not on the angle or the length of the inclined plane

(c) Remember also that the result of zero work for the normal force is quite general. The normal force ***N*** acting on any arbitrary body rolling or sliding on any kind of fixed surface never does work on the body, since this force is always perpendicular to the displacement.

EXAMPLE 5.2 Calculate the kinetic energy of each of the following:

(a) The Earth orbiting the Sun $m = 5.98 \cdot 10^{24}$ kg $v = 2.98 \cdot 10^{4}$ m/s

(b) Car $m = 1500$ kg $v = 27$ m/s

(c) World-class sprinter $m = 80$ kg $v = 10$ m/s

(d) Rifle bullet m = 0.01 kg v =1000 m/s

(e) Nitrogen molecule in air $m = 4.6 \cdot 10^{-26}$ kg v =500 m/s

Solution

Using $KE = mv^2/2$ yields the following interesting results:

(a) $2.66 \cdot 10^{33}$ J, (b) $5.47 \cdot 10^5$ J, (c) $4 \cdot 10^3$ J, (d) $5 \cdot 10^3$ J, (e) $5.8 \cdot 10^{-21}$ J

EXAMPLE 5.3 A 1000-kg elevator cage descends 400 m within a skyscraper.

(a) What is the work done by gravity on the elevator cage during this displacement?

(b) Assuming that the elevator cage descends at constant velocity, what is the work done by the tension of the suspension cable?

Solution

(a) With the x axis arranged vertically upward, the displacement is negative, Δx= -400 m; and the x component of the weight is also negative,

$$G_x = -mg = -9800 \text{ N}$$

Hence the work done by the weight is

$$W = G_x \Delta x = (-9800 \text{ N})(-400 \text{ m}) = 3.92 \cdot 10^6 \text{ J}$$

(b) For motion at constant velocity, the tension force must exactly balance the weight, so the net force $\boldsymbol{F}_{\text{net},x}$ is zero. Therefore, the tension force of the cable has the same magnitude as the weight, but the opposite direction: $T_x = mg$ = 9800 N

The work done by this tension force is then

$$W = T_x \Delta x = (9800 \text{ N})(-400 \text{ m}) = -3.92 \cdot 10^6 \text{ J}$$

This work is negative because the tension force and the displacement are in opposite directions. Gravity does work on the elevator cage, and the elevator cage does work on the cable.

COMMENTS: (a) Note again that the work done by gravity is completely independent of the details of the motion; the work depends on the total vertical displacement and on the weight, but not on the velocity or the acceleration of the motion. (b) Note that the work done by the tension is exactly the negative of the work done by gravity, and thus the net work done by both forces together is zero (we can also see this by examining the work done by the net force; since the net force $\boldsymbol{F}_{\text{net,x}}$ is zero, the net work is zero). However, the result for the work done by the tension depends implicitly on the assumptions made about the motion. Only for motion with no acceleration does the tension force remain constant at 9810 N. For instance, if the elevator cage were allowed to fall freely with the acceleration of gravity, then the tension would be zero; the work done by the tension would then also be zero, whereas the work done by gravity would still be $3.92 \cdot 10^6$ J.

EXAMPLE 5.4 What is the work performed in stretching a spring 0.1 m, when for stretching the spring $x_0 = 0.02$ m we need force $F_0 = 2500$ N?

Solution

$$F = -kx, k = \frac{F_0}{x_0}$$

The force is in the direction of the displacement so $\boldsymbol{F} \cdot \mathrm{d}\boldsymbol{x} = F\mathrm{d}x$

$$W = \int F\mathrm{d}x = k\int_0^{0.1} x\mathrm{d}x = k\frac{x^2}{2}\Bigg]_0^{0.1} = \frac{2500\ \mathrm{N}}{0.02\ \mathrm{m}} \cdot \frac{[(0.1\ \mathrm{m})^2 - 0]}{2} = 625\ \mathrm{J}$$

EXAMPLE 5.5 Place a block *m* = 3.0 kg at the top of a *h* = 3.4 m high frictionless incline. At the bottom of the incline the block encounters a spring with a constant of 400 N/m. No energy is lost to friction. How far is the spring compressed?

Solution

The potential energy at the top of the plane, which is the same as the kinetic energy at the bottom of the plane, goes into compressing the spring. The kinetic energy at the top of the plane is *mgh*

So: $mgh = \frac{kx^2}{2}$ or $x = \sqrt{\frac{2mgh}{k}} = \sqrt{\frac{2 \cdot 3 \cdot 9.8 \cdot 3.4\ \mathrm{J}}{400\ \mathrm{N/m}}} = 0.71\ \mathrm{m}$

EXAMPLE 5.6 An elevator car has a mass of 1 000 kg and is carrying passengers having a combined mass of 200 kg. A constant friction force of 4 000 N retards its motion.

(a) How much power must a motor deliver to lift the elevator car and its passengers at a constant speed of 3.00 m/s? (64.9 kW).

(b) What power must the motor deliver at the instant the speed of the elevator is *v* if the motor is designed to provide the elevator car with an upward acceleration of 1.00 m/s^2? (23.4 kW)

EXAMPLE 5.7 A block of mass 1.6 kg is attached to a horizontal spring that has a force constant of 10^3 N/m. The spring is compressed 2.0 cm and is then released from rest, (a) Calculate the speed of the block as it passes through the equilibrium position *x* = 0 if the surface is frictionless. (0.5 m/s) (b) Calculate the speed of the block as it passes through the equilibrium position if a constant frictional force of 4.0 N retards its motion from the moment it is released.

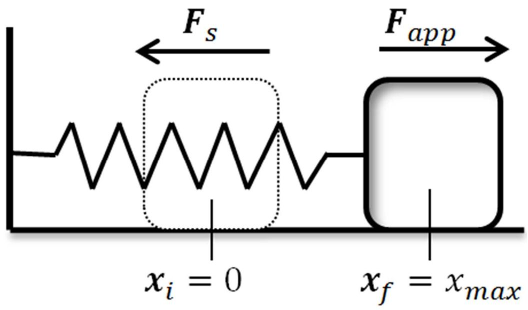


Fig. 5.3 A block of mass attached to a spring

Solution

(a) Block starts with zero initial speed and its initial position is $x_i = -2$ cm. The work done until it reaches $x_f = 0$ cm is:

$$W_s = \int_{-0.02}^{0} kx\mathrm{d}x = \frac{1}{2}kx^2\Big]_{-0.02}^{0} = \frac{1}{2}\cdot 10^3\mathrm{Nm}^{-1}\cdot(-2\cdot 10^{-2}\mathrm{m})^2 = 0.2\ \mathrm{J}$$

Using work-kinetic theorem, we get $W_s = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$, $0.2 = \frac{1}{2}(1.6\ \mathrm{kg})\ v_f^2 - 0$, hence $v_f = 0.5\ \mathrm{m/s}$.

(b)First calculate the lost ΔKE of kinetic energy because of friction:

$$\Delta KE = -F_{kr}d = -4\ \mathrm{N}\cdot 2\cdot 10^{-2}\ \mathrm{m} = -0.08\ \mathrm{J}$$

The final kinetic energy in the presence of friction is then

$$KE_f = 0.2\ \mathrm{J} - 0.08\ \mathrm{J} = 0.12\ \mathrm{J} = \frac{1}{2}mv_f^2$$

And hence $\quad v_f = \sqrt{2\cdot 0.12\ \mathrm{J}/m} = \sqrt{2\cdot 0.12\ \mathrm{J}/(1.6\ \mathrm{kg})} = 0.39\ \mathrm{m/s}$

EXAMPLE 5.8 Place a mass m on a track made up of a flat section, L, with coefficient of friction, μ, and two frictionless semicircular surfaces of radius R. Let the mass start from the top of one of the quartercircular and calculate where it comes to rest.

Solution

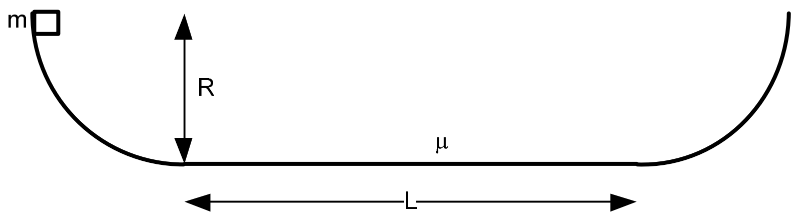


Fig. 5.4 A mass moving on a track

The initial potential energy is *mgR*. When the mass encounters the friction surface, this (potential) energy is dissipated in doing work to overcome friction. Assuming the energy lost due to friction in one traverse is less than the initial potential energy, the mass will rise to a height (on the opposite semicircle) R' dictated by the energy statement

$$mgR' = mgR - \mu mgL$$

After another traverse of the flat portion of the track the height will be dictated by

$$mgR'' = mgR - 2\mu mgL$$

and so on until all the original potential energy is dissipated.

EXAMPLE 5.9 In an amusement park roller coaster ride a car starts from rest at point A and races through a loop-the-loop. What is the minimum height *h* from which the car can start if it is not to leave the track at point B? The loop has radius *R*.

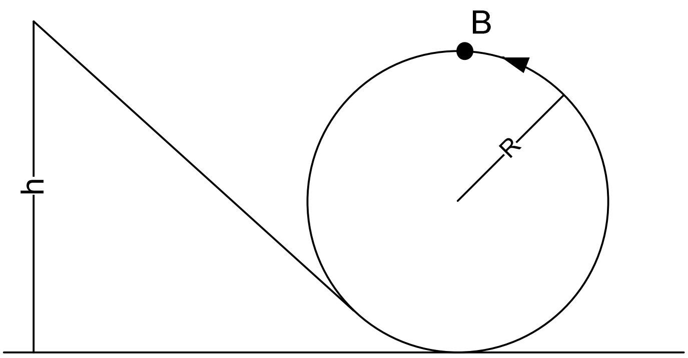


Fig. 5.5 A roller coaster ride

Solution

If the car is just about to leave the track at point B, the normal force exerted on the car by the track at this point is zero. The only force acting on the car is then *mg*, and this must provide the needed centripetal force to keep the car moving along the circular track. Thus

$$mg = \frac{mv^2}{R} \qquad \text{so} \qquad v^2 = Rg$$

We can find the speed *v* as a function of the starting elevation h by applying the conservation of energy principle. $mgh + 0 = mg2R + \frac{mv^2}{2}$, replacing $v^2 = Rg$

$$gh = g2R + gR/2, \quad h = 5R/2$$

EXAMPLE 5.10 Falling rock. If the original height of the rock is $y_1 = h = 3$ m, calculate the rock´s speed when it has fallen to 1 m above the ground.

Solution

One approach is to use the kinematic equations. Let us instead apply the principle of conservation of mechanical energy, assuming that only gravity acts on the rock. We choose the ground as our reference level (y = 0). At the moment of release (point 1) the rock´s position is y_1 = 3 m and it is at rest: v_1 = 0. We want to find v_2 when the rock is at position y_2 = 1 m.

$$mv_1^2/2 + mgy_1 = mv_2^2/2 + mgy_2$$

The m`s cancel out. Setting v_1 = 0 and solving for v_2^2 we find

$$v_2^2 = 2g(y_1 - y_2) = 2 \cdot 9.8\ \mathrm{m \cdot s^{-2}} \cdot (3 - 1)\ \mathrm{m} = 39.2\ \mathrm{m^2 s^{-2}}$$

$$v_2 = \sqrt{39.2}\ \ \mathrm{m \cdot s^{-1}} = 6.3\ \ \mathrm{m \cdot s^{-1}}$$

EXAMPLE 5.11 A compact car has a mass of 800 kg, and its efficience is rated at 18 % (that is, 18 % of the available fuel energy is delivered to the wheels). Find the amount of gasoline used to accelerate the car from rest to 27 m/s. Use the fact that the energy equivalent of 1 liter gasoline is $3{,}6 \cdot 10^7$ J.

Solution

The energy required to accelerate the car from rest to a speed v is equal to its final kinetic energy:

$$E_k = (1/2)mv^2 = (1/2)(800\ \mathrm{kg})(27\ \mathrm{m/s})^2 = 2.9 \cdot 10^5\ \mathrm{J}$$

If the engine were 100 % efficient, each liter of gasoline would supply $3{,}6 \cdot 10^7$ J of energy. Because the engine is only 18 % efficient, each liter delivers an energy of only $0.18 \cdot 3.6 \cdot 10^7\ \mathrm{J} = 6.5 \cdot 10^6$ J. Hence, the number V of liters used to accelerate the car is

$$V = \frac{2.9 \cdot 10^5\ \mathrm{J}}{6.5 \cdot 10^6\ \mathrm{J/L}} = 0.045\ \mathrm{L}$$

Let us estimate that it takes 10 s to achieve the indicated speed. The distance traveled during this acceleration is

$$\Delta x = \bar{v}\Delta t = \frac{v_i + v_f}{2}\Delta t = \frac{0 + 27}{2} 10 = 135\ \mathrm{m}$$

At a constant cruising speed of 27 m/s, 0.045 L of gasoline is sufficient to propel the car approximately 800 m, six times further. This demonstrates the extreme energy requirements of stop-and-start driving.

EXAMPLE 5.12 Car accelerating up a hill. A car of mass *m* is accelerating up a hill. The total resistive force in *N* is $f_t = (218 + 0.7v^2)$, where *v* is the speed in m/s. Determine the power the engine must deliver to the weels as a function of speed.

Solution

The forces on the car are shown in figure 5.6. Applying Newton´s second law to the motion along the road surface, we find

$$\sum F_x = F - f_t - mg\sin\theta = ma$$

$$F = ma + mg\sin\theta + f_t$$

$$F = ma + mg\sin\theta + (218 + 0.7v^2)$$

And the power required to move the car is

$$P = Fv = mav + mgv\sin\theta + 218v + 0.7v^3$$

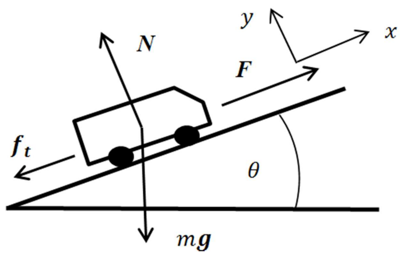


Fig. 5.6 A car moving uphill

The **term *mav*** is the power that the engine must deliver to accelerate the car. If the car moves at constant speed, this term is zero and the total power requirement is reduced. The **term $\boldsymbol{mgv\sin\theta}$** is the power required to provide a force to balance a component of the gravitational force as the car moves up the incline. This term would be zero for motion on a horizontal surface. The **term $\boldsymbol{218\,v}$** is the power to balance rolling friction, and the **term $\boldsymbol{0.7v^3}$** is the power needed against air drag.

If we take *m* = 1450 kg, *v* = 27 m/s, *a* = 1 m/s^2 and $\theta = 10°$, then the various terms in *P* are:

$$mav = (1450\text{ kg})(1\text{ m/s}^2)(27\text{ m/s}) = 39\text{ kW}$$

$$mgv\sin\theta = (1450\text{ kg})(9.8\text{ m/s}^2)(27\text{ m/s})(\sin 10°) = 67\text{ kW}$$

$$218\,v = 218(27\text{ m/s}) = 5.9\text{ kW}$$

$$0.7v^3 = 0.7(27\text{ m/s})^3 = 14\text{ kW}$$

So the total power required is 126 kW. Note that the power requirements for traveling at constant speed of 27 m/s on a horizontal surface are only 20 kW (the sum of the last two terms). Furthermore, if the mass were halved (the case of compact car), then the power required also is reduced by almost the same factor.

EXAPLE 5.13 The simple pendulum. A sphere of mass *m* attached to mass less cord of length *L* is released from rest when the cord makes an angle θ_A with the vertical and the pivot is frictionless. a) Find the speed of the sphere when it is at the lowest point B.

Solution

The only force that does work on the sphere is the gravitational force (the force of tension is always perpendicular to each element of the displacement and so does no work). Because the gravitational force is conservative, the total mechanical energy of the pendulum – Earth system is constant (in other words, we can classify this as an "energy conservation" problem) as the pendulum swings, continuous transformation between potential energy (PE) and kinetic energy (KE) occurs. At the instant the pendulum is released, the energy of the system is entirely potential energy. At point B the pendulum has kinetic energy, but the system has lost some potential energy. At C the system has regained its initial potential energy, and the kinetic energy of the pendulum is again zero.

We measure the y-coordinates of the sphere from the center of rotation, then from the principle of mechanical energy conservation:

$$y_A = -L\cos\theta_A, \qquad y_B = -L$$

$$PE_A = -mgL\cos\theta_A, \qquad PE_B = -mgL$$

$$KE_A + PE_A = KE_B + PE_B$$

$$0 - mgL\cos\theta_A = mv_B^2/2 - mgL$$

$$v_B = \sqrt{2gL(1-\cos\theta_A)}$$

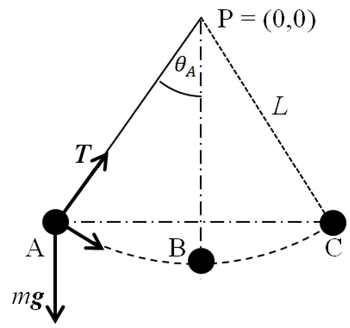


Fig.5.7 The simple pendulum

b) What is the tension T_B in the cord at point B?

Solution

Since the force of tension does no work, we cannot determine the tension using the energy method. To find T_B, we can apply Newton´s second law to the radial direction. First, recall that the radial acceleration a_r of a particle moving in a circle is equal to v^2/r directed toward the center of rotation. Since $r = L$, we obtain

$$\sum F_r = T_B - mg = ma_r = m{v_B}^2/L$$

Substituing v_B gives the tension at point B:

$$T_B = mg + 2mg(1-\cos\theta_A) = mg(3 - 2\cos\theta_A)$$

Note that the tension T_B in B is greater than the weight mg of the sphere.

Note, this equation gives the expected result that $T_B = mg$ when the initial angle $\theta_A = 0$.

EXAMPLE 5.14 A pendulum of length 2 m and mass 0.5 kg is released from rest when the cord makes an angle of 30° with vertical. Find the speed of the sphere and the tension in the cord when the sphere is at its lowest point. (2.29 m/s, 6.21 N).

EXAMPLE 5.15 On a frozen pond, a person kicks a 10 kg sled, giving it an initial speed of 2.2 m/s. How far does the sled move if the coefficient between the sled and ice is 0,1? (2.5 m)

EXAMPLE 5.16 A 3.00-kg crate slides down a ramp. The ramp is 1.00 m in length and inclined at an angle of θ = 30.0º as shown in next Figure. The crate starts from rest at the top, experiences a constant friction force F_f of magnitude 5.00 N, and continues to move a short distance on the horizontal floor after it leaves the ramp. (a) Use energy methods to determine the speed of the crate at the bottom of the ramp. (b) How far does the crate slide on the horizontal floor if it continues to experience a friction force of magnitude 5.00 N? (2.54 m/s; 1.94m)

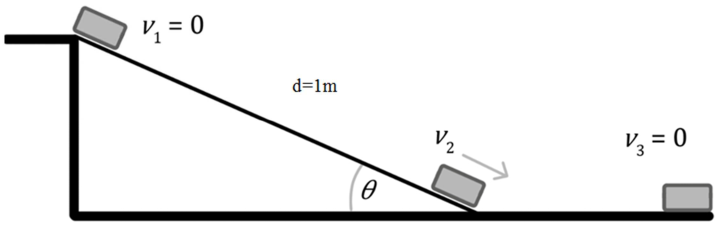


Fig. 5.8 A crate sliding down the ramp

6 GRAVITATIONAL FIELD

Newton's law of universal gravitation says: "Every particle in the universe attracts every other particle with a force directly proportional to the product of their masses and inversely proportional to the square of the distance between them. The direction of this force line is along the line joining the particles."

In vector form the **Newton's law of universal gravitation** is:

$$\boldsymbol{F} = -G\frac{m_1 m_2}{r^3}\boldsymbol{r}$$

$\boldsymbol{F}$ is the gravitational force exerted on the mass m_2 by the particle of mass m_1, $\boldsymbol{r}$ is the position vector of the particle 2 with respect to the particle 1, the minus sign indicates that the gravitational force has opposite direction to the position vector, it means it is directed towards the particle 1. The multiplication by the displacement vector does not change the inversely proportional dependence of the gravitational force to the square of the distance between the particles as the magnitude of the position vector can be divided with the cube of the position vector in the denominator.

The universal gravitational constant $G \approx 6{,}674 \cdot 10^{-11}$ N·m^2·kg^{-2}.

The property which characterizes the gravitational field itself is called **gravitational field strength** and is defined as:

$$\boldsymbol{E} = \boldsymbol{F}/m$$

$\boldsymbol{F}$ is the gravitational force acting on the particle of mass m. Using Newton's law of universal gravitation we can express the gravitational field created by the particle of mass M in a particular point in space with the position vector $\boldsymbol{r}$ as

$$\boldsymbol{E} = -G\frac{M}{r^3}\boldsymbol{r} \qquad ()$$

Gravitational potential energy is defined with respect to infinity, it means $E_p(r \to \infty) = 0$ and is defined with equation $E_p = -G\frac{m_1 m_2}{r}$

Gravitational field can be also defined with gravitational potential V with respect to any referential point, e.g. in infinity $V(r \to \infty) = 0$. Gravitational potential at certain point is $V = \frac{Ep}{m}$

EXAMPLE 6.1 The gravitational force between spherical objects and small objects can be modeled as particles having masses m_1 and m_2 separated by a distance *r*, is attractive and according to Newton's law of gravitation has a magnitude $F_g = Gm_1 m_2 / r^2$.

A satellite of mass *m* moves in a circular orbit around the Earth at a constant speed ***v*** at altitude *h* above the surface of Earth. Determine the speed of the satellite.

Solution

The only external force acting on the satellite is the force of gravity, which acts toward the center of the Earth and keeps the satellite in its circular orbit. Therefore,

$F_r = F_g = GM_E m / r^2$. From Newton's second law $F = ma$ we obtain

$mv^2 / r = GM_E m / r^2$. Solving for *v* gives

$v = \sqrt{GM_E / r} = \sqrt{GM_E / (r_E + h)}$, where r_E is the radius of Earth.

EXAMPLE 6.2 Geosynchronous satellite is one that stays above the same point on the Earth, which is possible only if it is above a point on the equator. Such satellites are used for TV and radio transmission, for weather forecasting, and as communication relays. Determine (a) the height above the Earth's surface such a satellite must orbit, (b) such a satellite's speed, and (c) compare to the speed of a satellite orbiting 200 km above Earth's surface.

Solution

To remain above the same point on Earth as the Earth rotates, the satellite must have a period of one day. We can apply Newton's second law, $F = ma$, where $a = v^2/r$ if we assume the orbit is circular.

(a) The only force on the satellite is the force of gravitation, so

$$GM_E m_S / r^2 = m_S v^2 / r, \qquad v = 2\pi r / T, \qquad T = 24 \cdot 3600 = 86400 \text{ s}$$

$$GM_E / r^2 = v^2 / r = (2\pi r / T)^2 / r = (2\pi)^2 r / T^2, \text{ or} \qquad r^3 = GM_E T^2 / (2\pi)^2$$

$$r = \sqrt[3]{GM_E T^2 / (2\pi)^2} = \sqrt[3]{6.67 \cdot 10^{-11} \text{ Nm}^2\text{kg}^{-2} \cdot 5.98 \cdot 10^{24} \text{ kg} \cdot (86400 \text{ s})^2 / (2\pi)^2}$$

$$r = 4.23 \cdot 10^7 \text{ m} = 42300 \text{ km}$$

Subtracting the Earth´s radius of 6378 km we receive, the geosynchronous satellite must orbit about 36,000 km (about 6 R_E) above the Earth´s surface.

(b) We solve for *v* in the satellite equation given in part (a):

$$v = \sqrt{GM_E / r} = 3070 \text{m/s} \qquad v = 2\pi r / T = 3070 \text{m/s},$$

(c) The equation in part (b) for v shows $v \approx \sqrt{1/r}$.

So for $r_h = r_E + h = 6380 + 200 = 6580$ km we get

$v_h = v\sqrt{r/r_h} = 3070\sqrt{42.300/6580}\,\text{m/s} = 7780\,\text{m/s}$

Note. The center of a satellite orbit is always at the center of the Earth, so it is not possible to have a satellite orbiting above a fixed point on the Earth at any latitude other than 0°.

EXAMPLE 6.3 Calculate the escape speed from Earth for a 5000 kg spacecraft, and determine the kinetic energy it must have at Earth´s surface in order to escape the Earth´s gravitational field.

Solution

$$KE_i + PE_i = KE_f + PE_f$$

$KE_f = 0$ because final velocity is zero, and $PE_f = 0$ because its final distance is infinity, so

$$\frac{mv_{esc}^2}{2} + \frac{-GM_E m}{R_E} = 0 + 0$$

$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{\frac{2\cdot 6.67\cdot 10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2\cdot 5.98\cdot 10^{24}\text{kg}}{6.37\cdot 10^6\text{m}}} = 1.12\cdot 10^4\ \text{m/s}$$

Kinetic energy of spacecraft: $KE = \frac{1}{2}mv_{esc}^2 = 3.13\cdot 10^{11}$ J

EXAMPLE 6.4 A satellite with a mass of 200 kg is placed in Earth orbit at a height of 200 km above the surface, (a) Assuming a circular orbit, how long does the satellite take to complete one orbit? (b) What is the satellite's speed? (c) What is the minimum energy' necessary to place this satellite in orbit (assuming no air friction)? ((a) 1.48 h, (b) 7.79 km/s, c) $6.43\cdot 10^9$ J)

7 MANY BODIES MECHANICS

When talking many bodies' mechanics we think of system of particles that is physically explored as one unit. The concept of **the center of mass (CM)** is helpful in understanding the behavior of a system of particles. For two particles of equal mass, the center of mass lies midway between them on the line joining them. For an object like a brick, the center of mass is at the geometrical center.

If a system consists of particles of mass m_1 at position r_1, m_2 at position r_2,...,and m_n at r_n, the position of the center of mass is defined to be

$$r_{CM} = \frac{m_1\boldsymbol{r}_1 + m_2\boldsymbol{r}_2 + \cdots m_n\boldsymbol{r}_n}{m_1 + m_2 + \cdots m_n} = \frac{m_1\boldsymbol{r}_1 + m_2\boldsymbol{r}_2 + \cdots m_n\boldsymbol{r}_n}{M} = \frac{1}{M}\sum\nolimits_1^n m_i\boldsymbol{r}_i$$

Here M is the total mass of the system. The x, y and z coordinates of the center of mass are

$$x_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i \qquad y_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i \qquad z_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i\ .$$

The center of mass of a system of particles with total mass *M* moves like an equivalent particle of mass *M* would move under the influence of the net external force on the system. $M\boldsymbol{a}_{CM} = \sum_i \boldsymbol{F}_{\text{ext},i}$

The total momentum $\boldsymbol{p}_{tot}$ of a system of particles is defined as:

$$\boldsymbol{p}_{tot} = \sum_i \boldsymbol{p}_i = \sum_i m_i\boldsymbol{v}_i = M\boldsymbol{v}_{CM}$$

Hence the total linear momentum of the system equals the total mass M multiplied by the velocity $\boldsymbol{v}_{CM}$ of the center of mass. In other words, the total linear momentum of the system is equal to that of a single particle of mass M moving with a velocity $\boldsymbol{v}_{CM}$.

Integrating $M\boldsymbol{a}_{CM} = \sum_i \boldsymbol{F}_{\text{ext},i}$

$$\int \sum_i \boldsymbol{F}_{\text{ext},i}\,\mathrm{d}t = \int M\boldsymbol{a}_{CM}\mathrm{d}t = M\int \frac{\mathrm{d}\boldsymbol{v}_{CM}}{\mathrm{d}t}\mathrm{d}t = M\int \mathrm{d}\boldsymbol{v}_{CM} = M\cdot\Delta\boldsymbol{v}_{CM} = \Delta\boldsymbol{p}_{tot} = \boldsymbol{I}$$

It means that the **total linear momentum of a system of particles is conserved if no net external force is acting on the system (** $\sum_i \boldsymbol{F}_{\text{ext},i} = 0$). It follows that for an isolated system of particles, both the **total momentum** and the velocity of the center of mass are constant in time. This statement is the **generalization of the law of conservation of momentum for a many-particle system**

Torque $\boldsymbol{\tau}$ is the **rotational quantity** analogous **to force**. For an object to acquire an angular acceleration, it must be subject to a non-zero net torque. Torque means "twist". The torque $\boldsymbol{\tau}$ due to a force $\boldsymbol{F}$ about a pivot P is

$$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F}$$

Where $\boldsymbol{r}$ is the position vector (a vector from the point from which torque is measured to the point where force $\boldsymbol{F}$ is applied) and the magnitude of the torque is

$$\tau = |\boldsymbol{\tau}| = rF\sin\theta$$

The angular momentum *L* of a particle relative to an axis through the origin O is defined by the cross product of the particle's instantaneous position vector $\boldsymbol{r}$ and its instantaneous linear momentum $\boldsymbol{p}$: $$\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$$

Now we can recognize that the concept of angular momentum is not going to be easy to work with. If I choose any point on line of momentum $\boldsymbol{p}$ vector, then the angular momentum relative to such point is zero (as both vectors are then in the same direction, θ is zero and $\sin\theta = 0$ as well). It is clear now that **angular momentum *L* is not an intrinsic property of a moving object, unlike momentum *p*, which is an intrinsic property. What the angular momentum is depends on the point of origin we choose. The total angular momentum of a system of particles is** $$\boldsymbol{L_{tot}} = \sum_{\boldsymbol{i}} \boldsymbol{r_i} \times m_{\boldsymbol{i}}\boldsymbol{v_i}$$

Time derivation of angular momentum $\frac{\mathrm{d}\boldsymbol{L}}{\mathrm{d}t} = \boldsymbol{\tau}_{ext}$ is analogous to $\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = \boldsymbol{F}_{ext}$ for translational motion. **If no external torque acts on a system, the angular momentum of the system remains constant:**

If $\boldsymbol{\tau}_{ext} = \boldsymbol{0}$ **then *L* =** const

This is the law of conservation of angular momentum *L*.

The moment of inertia $I = \sum_{\boldsymbol{i}} m_{\boldsymbol{i}} r_{\boldsymbol{i}}^{\boldsymbol{2}}$, $\boldsymbol{r}_i$ is the distance from the axis of rotation of i-th point with the mass m_i.

The total kinetic energy associated with rotational motion is

$$KE_R = \sum_i KE_{R_i} = \frac{1}{2}\sum_i m_i v_i^2 = \frac{1}{2}\sum_i m_i r_i^2\,\omega^2 = \frac{1}{2}\omega^2\sum_i m_i r_i^2 = \frac{1}{2}I\omega^2$$

$$KE = \frac{(I_{CM} + mR^2)\omega^2}{2} = \frac{I_{CM}\omega^2}{2} + \frac{mR^2\omega^2}{2} = \frac{I_{CM}\omega^2}{2} + \frac{mv^2}{2} = KE_R + KE_T$$

This is an important result. It states that **the kinetic energy of a rolling object is equal to the kinetic energy of translation of the center of mass (imagining all of the mass concentrated there) plus the kinetic energy of rotation about the CM.**

Collisions When two particles collide, the forces they exert on each other are much larger than any external forces acting. This allows us to assume that external forces are negligible; hence the momentum of the system remains constant. (We call such systems isolated). This means that the system's momentum just before a collision is the same as the momentum just after the collision. In a collision some kinetic energy of the particles is converted to heat, sound, plastic distortion, and so on. Such collisions are called inelastic collisions. Sometimes the loss in kinetic energy is negligible (as when two billiard balls collide). Such collisions are called elastic collisions, and for them the kinetic energy of the system is conserved before and after the collision. Of course, the kinetic energy of an individual particle can change, but the combined kinetic energy of both particles remains the same.

Inelastic collisions When two objects moving in the same direction, collide and then stick together, the collision is perfectly inelastic. If mass m_1 has speed v_1 and mass m_2 has parallel speed v_2 just before a perfectly inelastic collision (the particles stick together), the speed V just after the collision is determined by $m_1 v_1 + m_2 v_2 = (m_1 + m_2)V$ or

$$V = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

Elastic collisions Let us have a case of two masses moving in the same direction and then colliding (1D case, where signs take care of direction). If the collision is perfectly elastic, masses m_1 and m_2 can have different velocities V_1 and V_2 after the collision. Since **the momentum and the kinetic energy remains constant** and we have the case when the two masses move in the same direction before the collision

$$m_1 v_1 + m_2 v_2 = m_1 V_1 + m_2 V_2 \qquad \left(m_1 v_1^2\right)/2 + \left(m_2 v_2^2\right)/2 = \left(m_1 V_1^2\right)/2 + \left(m_2 V_2^2\right)/2 .$$

Equilibrium The requirements for a body to be in equilibrium are:

1. The vector sum of all external forces that act on the body must be zero
2. The vector sum of all external torques that act on the body must be zero

EXAMPLE 7.1 Show that the center of mass of a rod of mass M and length L lies midway between its ends, assuming the rod has a uniform mass per unit length.

Solution

$$x_{CM} = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_0^L x\lambda dx = \frac{\lambda}{M}\frac{x^2}{2}\Big|_0^L = \frac{\lambda L^2}{2M}$$

Substitute $\lambda = \dfrac{M}{L}$, $x_{CM} = L / 2$

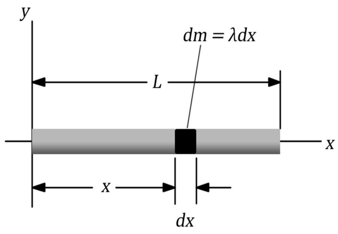


Fig. 7.1 A rod of mass M and length L

EXAMPLE 7.2 Suppose a rod is nonuniform such that its mass per unit length varies linearly with x according to the expression $\lambda = \alpha x$, where α is a constant. Find the x-coordinate of the center of mass as a fraction of L.

Solution

Because the mass per unit length is not constant in this case but is proportional to x, elements of the rod to the right are more massive than elements near the left end of the rod.

In this case, we replace dm by λdx, where $\lambda = \alpha x$.

$$x_{CM} = \frac{1}{M}\int x dm = \frac{1}{M}\int_0^L x\lambda dx = \frac{1}{M}\int_0^L x\alpha x dx = \frac{\alpha L^3}{3M}$$

The total mass of the rod: $M = \int dm = \int_0^L \alpha x dx = \dfrac{\alpha L^2}{2}$

Substituting $x_{CM} = \dfrac{2}{3}L$

EXAMPLE 7.3 Place a small ball of mass m_2 on top of a large ball of mass m_1 (m_1 » m_2). Drop the two simultaneously onto a floor. The result is impressive. The small ball takes off with big speed. We can calculate the height to which the ball would rise if the two are dropped from height h. We assume all collisions are elastic. (Hint: Imagine that first the big ball collides elastically with the floor, and then when it rebounds, it meets the falling ball that is right behind it.)

Solution

The big ball hits the floor with speed v_1 where conservation of energy during the fall yields

$$0 + m_1 gh = (1/2)\left(m_1 v_1^2\right) + 0 \qquad v_1 = \sqrt{2gh}$$

The ball bounces up with speed v_1 and collides with the small ball, whose velocity is $v_2 = -v_1$. After the collision the velocity of the small one is given by solution

If the collision is elastic the momentum and the kinetic energy remain constant

$$m_1 v_1 + m_2 v_2 = m_1 V_1 + m_2 V_2 \qquad \left(m_1 v_1^2\right)/2 + \left(m_2 v_2^2\right)/2 = \left(m_1 V_1^2\right)/2 + \left(m_2 V_2^2\right)/2 .$$

These two equations can be solved for the two unknown final speeds, V_1 and V_2

$$m_1\left(v_1^2 - V_1^2\right) = -m_2\left(v_2^2 - V_2^2\right) .$$

Factoring both sides of this equation: $m_1\left(v_1 - V_1\right)\left(v_1 + V_1\right) = -m_2\left(v_2 - V_2\right)\left(v_2 + V_2\right)$

Rearranging the first equation $m_1\left(v_1 - V_1\right) = -m_2\left(v_2 - V_2\right)$

Dividing the last two: $v_1 + V_1 = v_2 + V_2$ or $v_1 - v_2 = -\left(V_1 - V_2\right)$, where

v_1 - v_2 is the relative speed of particle 1 with respect to particle 2 before the collision, and $V_1 - V_2$ is the same quantity after the collision. It is an interesting result - the relative speed of one particle with respect to the other does not change in an elastic collision. We can solve this equation for V_2 and substitute it back into $m_1 v_1 + m_2 v_2 = m_1 V_1 + m_2 V_2$. Finally we get:

$$V_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 + \left(\frac{2m_2}{m_1 + m_2}\right) v_2 \qquad V_2 = \left(\frac{2m_1}{m_1 + m_2}\right) v_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_2$$

where $m_1 >> m_2$ $\quad V_2 \cong 2v_1 - v_2 = 2v_1 - \left(-v_1\right) = 3v_1 = 3\sqrt{2gh}$

Applying conservation of energy to the rising small ball gives us the height h to which the small ball rises. $\left(m_2 V_2^2\right)/2 + 0 = 0 + m_2 g h^/$ $\quad h^/ = \left(3\sqrt{2gh}\right)^2 / (2g) = 9h$!!!

EXAMPLE 7.4. The blade of a circular saw is initially rotating at 7000 revolutions per minute. Then the motor is switched off, and the blade comes to a stop in 8 s. What is the average angular acceleration? (-91.6 radians/s^2)

EXAMPLE 7.5 An automobile accelerates uniformly from 0 to 80 km/h in 6 s. The wheels of the automobile have a radius of 0.3 m. What is angular acceleration of the wheels? Assume that the wheels roll without slipping. (12 radians/m^2)

EXAMPLE 7.6 The large centrifuge has an arm of length 8.8 m. When rotating at 175 revolutions per minute, what is the speed of the end of this arm and what is the centripetal acceleration? (v = 1.6 $\cdot 10^2$ m/s; a_c = 2.9 $\cdot 10^3$ m/s^2)

EXAMPLE 7.7 A uniform rod of length L and mass M is free to rotate on a frictionless pin passing through one end. The rod is released from rest in the horizontal position.

(a) What is its angular speed when the rod reaches its lowest position?

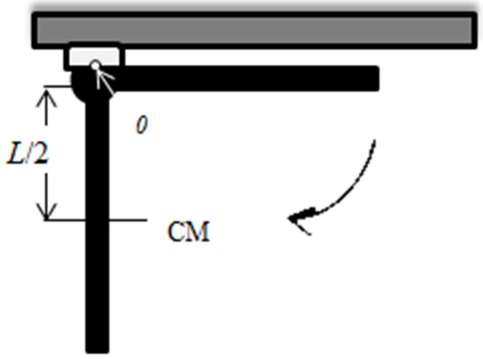


Fig.7.2 A uniform rod of length L and mass M free to rotate on a frictionless pin

Solution

We choose the configuration in which the rod is hanging straight down as the reference configuration for gravitational potential energy and assign a value of zero for this configuration. When the rod is in the horizontal position, it has no rotational kinetic energy. The potential energy of the system in this configuration relative to the reference configuration is $MgL/2$ because the center of mass of the rod is at a height $L/2$ higher than its position in the reference configuration. When the rod reaches its lowest position, the energy of the system is entirely rotational energy $\frac{1}{2}I\omega^2$, where $\boldsymbol{I}$ is the moment of inertia of the rod about an axis passing through the pivot. Using the isolated system (energy) model, write a conservation of mechanical energy equation for the system: $KE_f + PE_f = KE_i + PE_i$

By substituting we get $\frac{1}{2}I\omega^2 + 0 = 0 + \frac{1}{2}Mgl$

Solving the equation for ω and using $I = \frac{1}{3}ML^2$ for the rod

$$\omega = \sqrt{\frac{MgL}{I}} = \sqrt{\frac{3g}{L}}$$

(b) Determine the tangential speed of the center of mass and the tangential speed of the lowest point on the rod when it is in the vertical position.

Solution

Using the result from part (a)

$$v_{CM} = r\omega = \frac{L}{2}\omega = \frac{1}{2}\sqrt{3gL}$$

Because *r* for the lowest point on the rod is twice what it is for the center of mass

$$v = 2v_{CM} = \sqrt{3gL}$$

EXAMPLE 7.8 A 75 kg hockey player traveling at 12 m/s collides with a 90 kg player traveling, at right angles to the first, at 15 m/s . The players stick together. Find their resultant velocity and direction. Assume the ice surface to be frictionless.

Solution

This problem can be analyzed by conservation of momentum. Calculate the momenta and draw a vector diagram.

$p_1 = 75\,kg\,(12\,m/s) = 900\,kg \cdot m/s$

$p_2 = 90\,kg\,(15\,m/s) = 1350\,kg \cdot \text{m/s}$

The angle of the two hockey players is:

$\tan\theta = 1350/900 = 1.5, \quad or\ \theta = 56°$

And the resulting momentum is:

$p = \sqrt{1350^2 + 900^2} = 1620\,\text{kg} \cdot \text{m/s}$

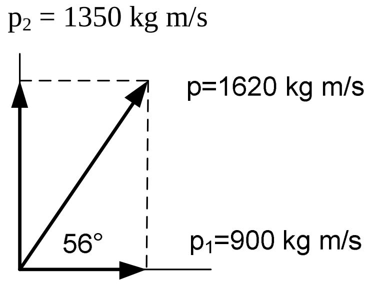


Fig. 7.3 Momentum vector diagram

The players move off with velocity $v = p/(m_1 + m_2) = 9.83\,\text{m/s}$ at an angle of 56° to the original direction of the 75 kg.

The second solution: A more formal approach is to write a conservation of momentum statement equating the total (vector) momentum before the collision to the total (vector) momentum after the collision. Take the plus ***i*** direction as the initial direction of the first player and the plus ***j*** direction as the original direction of the second player. Using the numbers already calculated:

$[900i + 1350\,j]\text{kg} \cdot \text{m/s} = (165\,kg)v$ or $v = [5.45i + 8.18\,j]\text{m/s}$

EXAMPLE 7.9 A ballistic pendulum, a device for measuring the speed of a bullet, consists of a block of wood suspended by cord. When the bullet is fired into the block, the block is free to rise. How high does a 5.0 kg block rise when a 12 g bullet traveling at 350 m/s is fired into it?

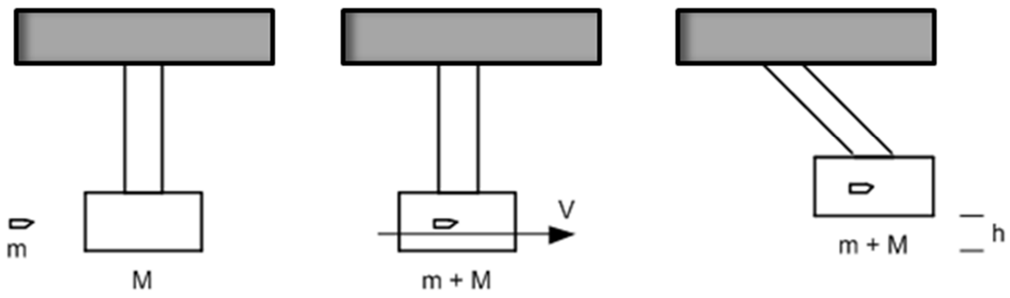


Fig. 7.4 A ballistic pendulum

Solution

The collision between the bullet and the block is clearly inelastic (the bullet comes to rest in the block). Part of the kinetic energy of the bullet goes into friction as the bullet burrows its way into the block. Therefore mechanical energy is not conserved.

Because the collision is inelastic, apply conservation of momentum of the collision. Before the collision, all the momentum is in the mv of the bullet. After the collision, the momentum is in the $(m + M)\,V$ of the block and bullet. We assume that the bullet comes to rest (transfer all its momentum) before there is appreciable motion of the bullet-block combination.

$mv = (m + M)V$

After the collision, the rise of the block is determined by energy analysis. The kinetic energy of the block goes into potential energy. $(m+M)V^2/2 = (m+M)gh$ or $V^2 = 2gh$

Substituting for V from $mv = (m + M)\,V$

$\left(\frac{m}{m+M}\right)^2 v^2 = 2gh$, so $v = \frac{m+M}{m}\sqrt{2gh}$ or $h = \frac{v^2}{2g}\left(\frac{m}{m+M}\right)^2$

gives the relation between the velocity of the bullet and the height the block and bullet rise.

For this problem $h = \frac{(350\,m/s)^2}{2\times 9.8m/s^2}\left(\frac{0.012}{5.012}\right)^2 = 3.6\,cm$

In this problem the 0.012 can be neglected in comparison to 5.0. This is not always the case so we write $m + M$ as 5.012 as a reminder to include both $m + M$ in the calculation.

EXAMPLE 7.10 A 6.0 g bullet is fired horizontally into a 2.8 kg block resting on a horizontal surface with coefficient of friction 0.30. The bullet comes to rest in the block, and the block slides 0.65 m before coming to a stop. What is the velocity of the bullet?

Solution

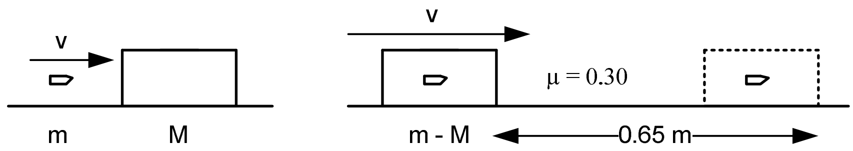


Fig. 7.5 A bullet fired into a block

Assume the bullet comes to rest in the block before the block moves appreciably and that all the momentum in the bullet is transferred to the bullet-block combination.

$$mv = (m+M)V$$

Once the bullet-block combination is moving at V, the kinetic energy, (1/2) $(m + M)V^2$, goes into work to overcome friction $\mu(m + M)gx$

$$(m+M)V^2/2 = \mu(m+M)gh, \qquad \text{or} \qquad V^2 = 2\mu g h.$$

Substituting $\left(\dfrac{m}{m+M}\right)^2 v^2/2 = \mu gh$

Or $\quad v = \dfrac{m+M}{m}\sqrt{2\mu gh} = \dfrac{2.806}{0.006}\sqrt{2\times 0.30(9.81\,\text{m/s}^2)0.65\,\text{m}} = 914\,\text{m/s}$

EXAMPLE 7.11 A father of mass m_f and his daughter of mass m_d sit on opposite ends of a seesaw at equal distances from the pivot at the center. The seesaw is modeled as a rigid rod of mass M and length l, and is pivoted without friction. At a given moment, the combination rotates in a vertical plane with an angular speedω.

(a) Find an expression for the magnitude of the system's angular momentum L.

Solution

Ignore any movement of arms or legs of the father and daughter and model them both as particles. The system is therefore modeled as a rigid object. The moment of inertia of the system equals the sum of the moments of inertia of the three components: the seesaw and the two individuals. We will use the moment of inertia of the rod calculated to the horizontal axis passing through the center $I = \dfrac{1}{12}Ml^2$ and for each person $I = mr^2$, $(r = {}^{l}/_{2})$.

The total momentum is then:

$$I = \frac{1}{12}Ml^2 + m_f\left(\frac{l}{2}\right)^2 + m_d\left(\frac{l}{2}\right)^2$$

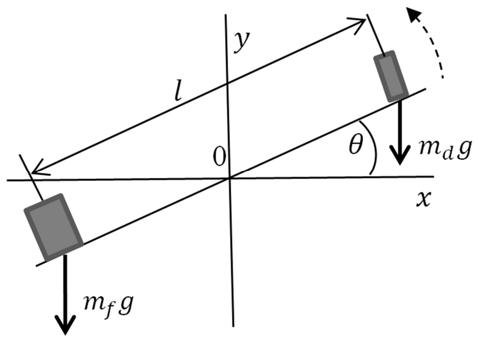


Fig. 7.6 Seesaw

The magnitude of the angular momentum L of the system is then

$$L = I\omega = \frac{l^2}{4}\left(\frac{M}{3} + m_f + m_d\right)\omega$$

(b) Find an expression for the magnitude of the angular acceleration $\boldsymbol{\alpha}$ of the system when the seesaw makes an angle θ with the horizontal.

Solution

Generally, fathers are more massive than daughters, so the system is not in equilibrium and has an angular acceleration. We identify the system as non-isolated because of the external torque associated with the gravitational force, the axis of rotation to be z-axis. To find the angular acceleration of the system at any angle, we first calculate the net torque on the system and then use $\sum \tau_{ext} = I\alpha$ from the rigid object under a net torque model to obtain an expression for α.

$$\tau_f = m_f g \frac{l}{2}\cos\theta$$

$$\tau_d = -m_d g \frac{l}{2}\cos\theta$$

$$\tau_{ext} = \tau_f + \tau_d = \frac{1}{2}(m_f - m_d)gl\cos\theta$$

And hence $$\alpha = \frac{\sum \tau_{ext}}{I} = \frac{2(m_f - m_d)g\cos\theta}{l\left(\frac{M}{3} + m_f + m_d\right)}$$

EXAMPLE 7.12 A star rotates with a period of 30 days about an axis through its center. The period is the time interval required for a point on the star's equator to make one complete revolution around the axis of rotation. After the star undergoes a supernova explosion, the stellar core, which had a radius of 10^4 km, collapses into a neutron star of radius 3.0 km. Determine the period of rotation of the neutron star.

Solution

Let us assume that during the collapse of the stellar core, (1) no external torque acts on it, (2) it remains spherical with the same relative mass distribution, and (3) its mass remains

constant. We consider the star as an isolated system in terms of angular momentum. We do not know the mass distribution of the star, but we have assumed the distribution is symmetric, so the moment of inertia can be expressed as kMR^2, where k is some numerical constant.
(k =2/5 for a solid sphere and k = 2/3 for a spherical shell.)
Let's T is the period, with T_i being the initial period of the star and T_f being the period of the neutron star. The star's angular speed is given by $\omega = \frac{2\pi}{T}$.

$$I_i \omega_i = I_f \omega_f$$

By substitution for ω and I we get $$kMR_i^2 \left(\frac{2\pi}{T_i} \right) = kMR_f^2 \left(\frac{2\pi}{T_f} \right)$$

Hence:

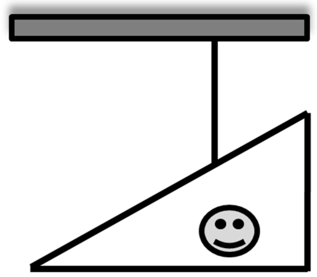
$$T_f = \left(\frac{R_f}{R_i} \right)^2 T_i$$

using the given numbers we get $T_f = 0.23$ s

EXAMPLE 7.13 Steel wire, length of 20 cm was bent at a right angle such that the sides of the wire are the same size and are oriented in the *x, y* axis direction. Bending point is identical to the beginning of the coordinate system. Determine the center of mass of the bent wire in the given coordinate system. (2, 5 cm; 2, 5 cm; 0 cm)

EXAMPLE 7.14 The system of bodies is formed by two spheres of different mass m_1 = 3 kg and m_2 = 12 kg. If you know that the first sphere moves with the speed $v_1 = 5$ m/s and the other is at rest, determine the speed of movement of the center of mass of the system. (v = 1m/s)

EXAMPLE 7.15 You have been asked to hang a metal sign from a single vertical string. The sign has the triangular shape shown in Figure 7.7.



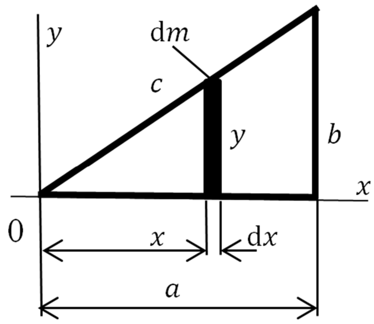


Fig.7.7 Hanging metal sign.

The bottom of the sign is to be parallel to the ground. At what distance from the left end of the sign should you attach the support string? ($x_{CM} = \frac{2}{3}a$)

EXAMPLE 7.16 A 1500-kg car traveling east with a speed of 25.0 m/s collides at an intersection with a 2500-kg truck traveling north at a speed of 20.0 m/s as shown in Fig.7.8.

Find the direction and magnitude of the velocity of the wreckage after the collision, assuming the vehicles stick together after the collision.

(v =15.6 m s^{-1}, $\theta = 53.1°$)

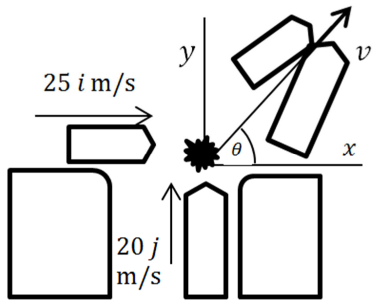


Fig.7.8 Cars' collision

8 EQUILIBRIUM AND ELASTICITY

Real materials are not perfectly rigid. When subjected to forces, they deform. If a substance deforms when subjected to a force, but returns to its initial shape when the force is removed, the substance is elastic.

The **stress** applied to a material is the force per unit area applied to the material:

$$\text{Stress} = F/A$$

It is measured in Nm^{-2} or pascals (Pa)

Strain is the ratio of extension to original length, it has no units as it is a ratio of two lengths measured in metres.

$$\text{Strain} = \Delta L / L$$

An **elastic modulus**, or **modulus of elasticity**, is a number that measures an object or substance's resistance to being deformed elastically (i.e., non-permanently) when a force is applied to it.

Elastic modulus $= (stress)/(strain)$

Tensile stress (or tension) is the stress state leading to expansion. The volume of the material stays constant. When equal and opposite forces are applied on a body, then the stress due to this force is called tensile stress. Young's modulus E is defined as :

$$E = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F / A}{\Delta L / L}, \quad \text{the SI unit is Pa}$$

The shear modulus describes an object's tendency to shear (the deformation of shape at constant volume) when acted upon by opposing forces. Suppose a piece of material, in the form of a rectangular block (like a brick), has one face fixed and a force $\boldsymbol{F}$ applied to the opposite face, of area A. If the two faces are separated by distance h and the sheared face moves Δx, the shear modulus is defined:

$$S = \frac{\text{shear stress}}{\text{sheer strain}} = \frac{F / A}{\Delta x / h}, \quad \text{the SI unit is Pa}$$

The bulk modulus B describes volumetric elasticity, or the tendency of an object to deform in all directions when uniformly loaded in all directions; it is defined as volumetric stress over volumetric strain, and is the inverse of compressibility. The bulk modulus is an extension of Young's modulus to three dimensions.

$$B = \frac{volume \quad stress}{volume \quad strain} = -\frac{F \ / \ A}{\Delta V \ / V} = -\frac{p}{\Delta V \ / V}, \qquad \text{the SI unit is Pa}$$

The negative sign is inserted so that B is a positive number because ΔV is negative due to a positive pressure. In some tables of data the inverse of B, called the compressibility, is tabulated. A large bulk modulus means that it is difficult to compress the material, whereas a large compressibility means, that it is easy to compress the material.

EXAMPLE 8.1 Place a 7.0 m uniform, 150 N ladder against a frictionless wall at an angle of 75°. What are the reaction forces at the ground and wall and the minimum coefficient of friction of the ground?

Solution. Figure 8.1 shows the ladder with the 150 N acting down at the center of the ladder and the sides of the triangle formed by the ladder, wall, and ground.

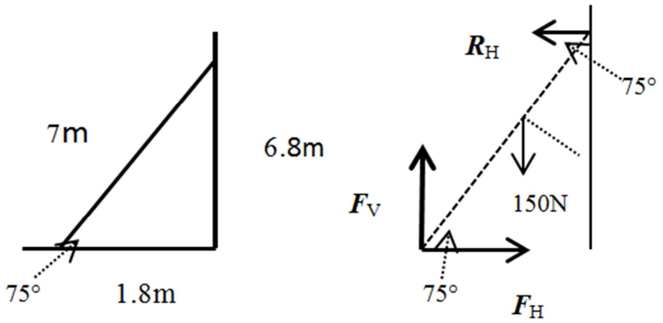


Fig.8.1 Free body diagram

$\sum F_x = 0$; $F_H = R_H$

$\sum F_Y = 0$; $F_V = 150N$

$\sum \tau = 0$; the torque on the ladder is taken about the point where the ladder contacts the ground. This choice eliminates two variables from torque statement. As torque is the component of the force at right angles to the lever arm times that lever arm.

Hence

$(150N \cos 75°)3.5\,\text{m} = (R_H \cos 15°)7{,}0\,\text{m}$

As: $F_H = R_H$ and $F_H = \mu F_V$

$\mu = 0.19$

EXAMPLE 8.2 To moor a ship, a sailor wraps a rope around a bollard (a cylindrical post). By pulling with a small force $\boldsymbol{T_1}$, he can control a much larger tension $\boldsymbol{T_2}$ on the end of the rope attached to the ship because of the friction between the rope and the bollard. The coefficient of friction between the rope and the bollard is μ = 0.2. If the sailor pulls with 400 N, how many turns are needed if he is to exert a force of 24,000 N on the ship?

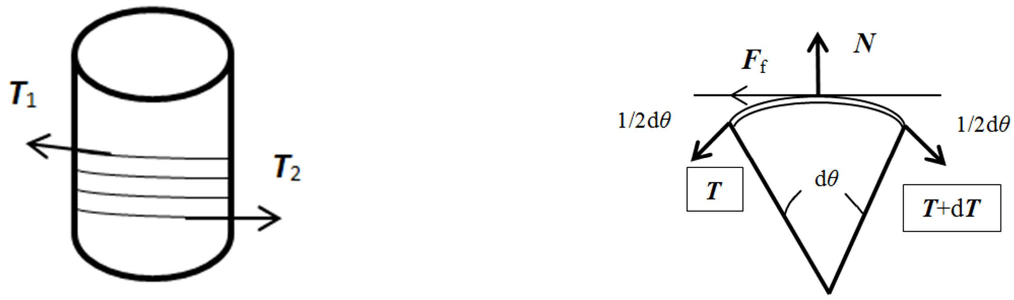


Fig.8.2 The bollard and the free body diagram

Solution

If we look at a small segment of rope that subtends a small angle dθ. Because of friction the tension at one end is $\boldsymbol{T}$ and at the other end slightly larger, $\boldsymbol{T} + \mathrm{d}\boldsymbol{T}$. Applying $\sum F_y = 0$:

$$N - T\sin(\mathrm{d}\theta/2) - (T + \mathrm{d}T)\sin(\mathrm{d}\theta/2) = 0$$

Using small angle approximation $\sin(\mathrm{d}\theta/2) \sim \mathrm{d}\theta/2$ and neglecting the very small term $\mathrm{d}T\,\mathrm{d}\theta/2$ gives: $N = T\mathrm{d}\theta$

The friction force is: $F_f = \mu N = \mu T\mathrm{d}\theta$

Applying $\sum F_x = 0$: $(T + \mathrm{d}T)\cos(\mathrm{d}\theta/2) - F_f - T\cos(\mathrm{d}\theta/2) = 0$

Small angle approximation $\cos(\mathrm{d}\theta/2) \sim 1$ gives:

$$\mathrm{d}T = F_f = \mu T\mathrm{d}\theta, \quad \text{or} \quad \frac{\mathrm{d}T}{T} = \mu\mathrm{d}\theta$$

If the tensions at the two ends are T_1 and T_2, then

$$\int_{T_1}^{T_2}\frac{\mathrm{d}T}{T} = \mu\int_{\theta_1}^{\theta_2}\mathrm{d}\theta \quad \text{or} \quad \ln\frac{T_2}{T_1} = \mu(\theta_2 - \theta_1)$$

$$(\theta_2 - \theta_1) = \frac{1}{\mu}\ln\frac{T_2}{T_1} = \frac{1}{0.2}\ln\frac{24{,}000}{400} = 5 \cdot \ln 60 = 20.5\ \text{rad}$$

$$\frac{20.5\ \text{rad}}{2\pi\ \ \text{rad/rev}} = 3.3\ \text{rev}$$

In this way sailors control the ship, they can even slowly release it, and then the force on the other side will start to move. But you cannot lift things with it. The frictional forces for the rope will flip over. In other words, what is now T_1 in our calculations will become T_2.

EXAMPLE 8.3 To span the space architects used round (semicircular) arch and in about A.D. 1100 the pointed arch came into use and became the hallmark of the great Gothic cathedrals. To make an accurate analysis of a stone arch is quite difficult in practice. But if we make some simplifying assumptions, we can show why the horizontal component of the force at the base is less for a pointed arch than for a round one. Figure 8.3 shows a round arch and a pointed arch, each with an 8 m span. The height of the round arch is thus 4.0 m. Pointed arch is larger and has been chosen to be 8.0 m. Each arch supports a weight of $12.0{\cdot}10^4$ N Calculate the horizontal force for each arch.

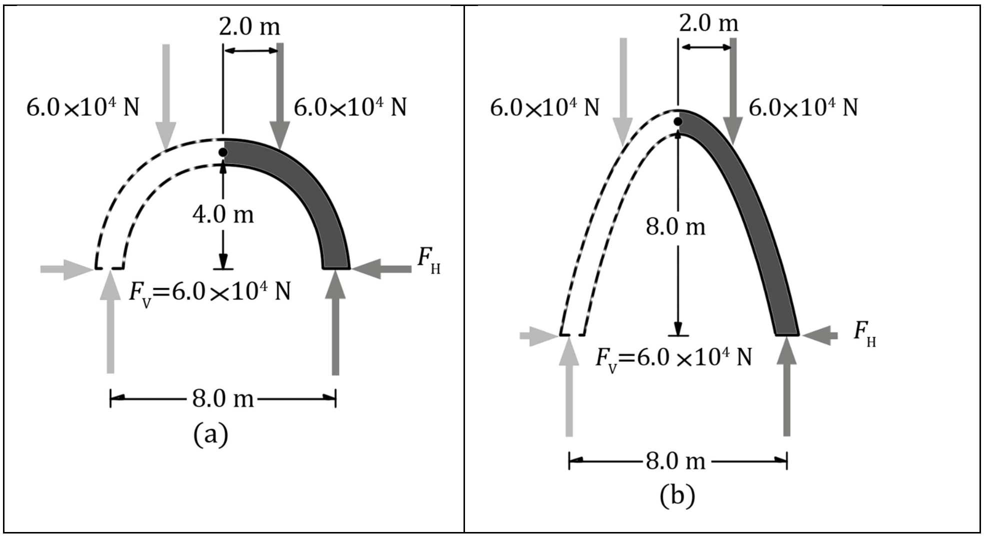


Fig.8.3 Forces in round (a) and pointed (b) arch

Solution

For simplicity let us divide the arches into two parts (each $6{\cdot}10^4$ N). For the arch to be in equilibrium, each of the supports must exert an upward force of $6{\cdot}10^4$ N. Each support also exerts a horizontal force, $\boldsymbol{F}_{\mathrm{H}}$, at the base of the arch, and it is this we want to calculate. We focus only on the right half of each arch. We set equal to zero the total torque calculated about the apex of the arch due to the forces exerted on that half arch, as if there were a hinge at the apex.

For the round arch, the torque equation $\sum M = 0,$ is as follows:

$$(4\ \mathrm{m})(6 \cdot 10^4\ \mathrm{N}) - (2\ \mathrm{m})(6 \cdot 10^4\ \mathrm{N}) - (4\ \mathrm{m})(\boldsymbol{F}_H) = 0$$

Thus $\boldsymbol{F}_{\mathrm{H}} = 3 \cdot 10^4$ N for the round arch.

For the pointed arch, the torque equation is:

$$(4\text{ m})(6 \cdot 10^4\text{ N}) - (2\text{ m})(6 \cdot 10^4\text{ N}) - (8\text{ m})(\boldsymbol{F}_H) = 0$$

For the pointed arch we get $\boldsymbol{F}_{\mathrm{H}} = 1{,}5 \cdot 10^4$ N and this is only half as much as for the round arch.

EXAMPLE 8.4 The Leaning Tower of Pisa is 55 m tall and about 7.0 m in diameter. The top is 4.5 m off center. Is the tower in stable equilibrium? If so, how much farther can it lean before it becomes unstable? Assume the tower is of uniform composition.

Solution

We know that an object whose center of gravity-CG is above its base of support will be stable if a vertical line projected downward from the CG falls within the base of support. For the tower, the base of support is a circle of radius 3.5 m. If the top is 4.5 m off center, then the CG will be 2.25 m off center, and a vertical line downward from the CG will be 2.25 m from the center of the base. Thus the tower is in stable equilibrium.

To be unstable, the CG has to be more than 3.5 m off center, and thus the top must be more than 7.0 m off center. Thus the top will have to lean 2.5 m further to reach the verge of instability.

EXAMPLE 8.5 Four bricks arc to be stacked at the edge of a table, each brick overhanging the one below it, so that the top brick extends as far as possible beyond the edge of the table. (a) To achieve this, show that successive bricks must extend no more than (starting at the top) 1/2, 1/4, 1/6, and 1/8 of their length beyond the one below (Fig. 8.7). (b) Is the top brick completely beyond the base? (c) Determine a general formula for the maximum total distance spanned by *n* bricks if they are to remain stable, (d) A builder wants to construct a corbeled arch based on the principle of stability discussed in (a) and (c) above. What minimum number of bricks, each 0.30 m long, is needed if the arch is to span 1.0 m?

Solution

(a) The maximum distance for the first brick (1) to remain on the second brick will be when the CM of the first brick will be directly over the edge of the second brick. We get $x_1 = L/2$

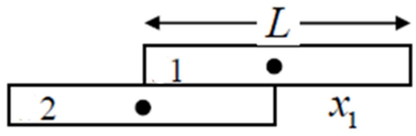


Fig.8.4 Maximum span for 2 bricks

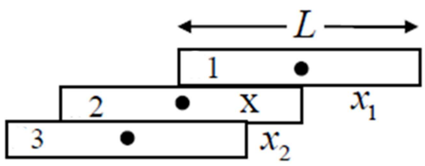


Fig.8.5 Maximum span for 3 bricks

The maximum distance for the top two bricks to remain on the third brick will be reached when the center of mass of the top two bricks is directly over the edge of the third brick. The CM of the top two bricks is in the middle of their CM`s and it means $L/4$ from the right edge of the second brick. Thus $x_2 = L/4$.

Notice that each time the x-coordinate of CM of upper n bricks is over the edge of the brick number (n+1) or the base.

So the maximum distance for the top three bricks to remain on the brick number 4 will be reached when the center of mass of the top three bricks is directly over the edge of brick number 4. The CM of the top three bricks is found relative to the center of brick 3 by:

$\mathrm{CM}_3 = \frac{m \cdot 0 + 2 \cdot m \cdot (L/2)}{3m} = L/3,$ or L/6 from the right edge of the brick 3. So $x_3 = L/6$,

where m is the mass of one brick.

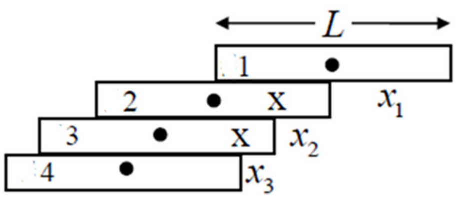


Fig.8.6 Maximum span for 4 bricks

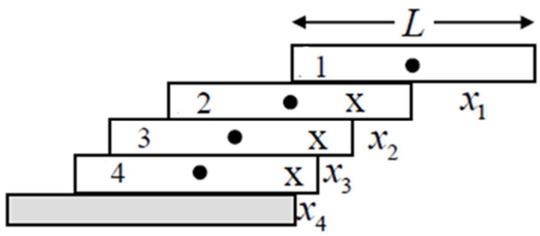


Fig.8.7 Maximum span for 4 bricks over the edge of the table

For four bricks to remain on the table, the CM of the four bricks has to be directly over the edge of the table. The CM found relative to the center of brick 4 will be:

$\mathrm{CM}_4 = \frac{m \cdot 0 + 3 \cdot m \cdot (L/2)}{4m} = 3L/8,$ or $L/8$ from the right edge of the brick 4 and $x_4 = L/8$.

From the results above, the distance from the edge of the table to the right edge of brick 1 is:

$$x_4 + x_3 + x_2 + x_1 = (L/8) + (L/6) + (L/4) + (L/2) = 25L/24 > L$$

Since this distance is greater than L, the first brick is completely beyond the edge of the table.

(c) The x-coordinate of CM of n bricks relative to the center of brick number n can be according to previous explanation calculated as:

$$\mathrm{CM}_n = \frac{m \cdot 0 + (n-1) \cdot m \cdot (L/2)}{n \cdot m} = L\frac{n-1}{2 \cdot n}$$

So the span x_i is: $x_i = \frac{L}{2} - \mathrm{CM}_n = \frac{L}{2} - L\frac{n-1}{2\cdot n} = L\frac{n-(n-1)}{2\cdot n} = L\frac{1}{2\cdot n}$

The general formula for the total distance spanned by n bricks over an edge is:

$$x_1 + x_2 + x_3 + \cdots\cdots + x_n = \left(\frac{L}{2}\right) + \left(\frac{L}{4}\right) + \left(\frac{L}{6}\right) + \cdots\cdots \left(\frac{L}{2n}\right) = \sum_{i=1}^{n}\frac{L}{2i} = \frac{L}{2}\sum_{i=1}^{n}\frac{1}{i}$$

We got so called harmonic series.

(d)The arch is to span 1.0 m, so the span from one side will be 0.50 m. We have to solve

$$\sum_{i=1}^{n}\frac{L}{2i} = \sum_{i=1}^{n}\frac{0.3}{2i} \geq 0.5$$

Tab. 8.1 Partial sums of foregoing harmonic series

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Σ	0,15	0,225	0,275	0,313	0,343	0,368	0,389	0,408	0,424	0,439	0,453	0,465	0,477	0,488	0,498	0,507

The table of partial sums of this harmonic series which we calculated for $L = 0.3$ and $i = 1$ to 16. Evaluation of this table shows that 15 bricks will span the distance of 0.498 m and 16 bricks will span a distance of 0.507 m. Therefore we have to take 16 bricks for each half-span, plus 1 brick as the base on each side, which means the total of 34 bricks.

EXAMPLE 8.6 A steel beam used in the construction of a bridge is 10.2 m long with a cross-sectional area A of 0.12 m^2. It is mounted between two concrete abutments with no room for expansion. When the temperature raises 10º C, such a beam will expand in length by 1.2 mm if it is free to do so. What force must be exerted by the concrete to keep this expansion from happening? Young's modulus for steel is $2\cdot10^{11}$ N/m^2.

Solution

$$F = Y\left(\frac{\Delta L}{L}\right)A = (2\cdot 10^{11}\ \mathrm{N/m^2})\left(\frac{1.2\cdot 10^{-3}\ \mathrm{m}}{10.2\ \mathrm{m}}\right)(0.12\ \mathrm{m^2}) = 2.8\cdot 10^{6}\ \mathrm{N}$$

This force will crack the concrete. The forces involved in thermal expansion can be huge, which is why it is necessary to leave expansion space in joints in large structures like bridges and buildings. And to make dilatation arcs in long tubes for heating, cooling etc.

EXAMPLE 8.7 Is the Young's Modulus for a bungee cord smaller or larger than that for the ordinary rope?

Solution

The Young's modulus for a bungee cord is much smaller than that for ordinary rope. The bungee cord stretches relatively easily, compared to ordinary rope. From the equation $E = \frac{F/A}{\Delta L/L}$ we can see that the value of Young's modulus is inversely proportional to the relative change in length of a material under a tension. Since the change in length of a bungee cord is much larger than that of an ordinary rope if other conditions are identical (stressing force, unstretched length, cross-sectional area of rope or cord), it must have a smaller Young's modulus.

EXAMPLE 8.8 A certain person's biceps muscle has a maximum cross-sectional area of 12 cm^2. What is the stress in the muscle if it exerts a force of 300 N? ($2.5 \cdot 10^5$ N/m^2)

EXAMPLE 8.9 A 500 kg mass is hung from a 3 m steel wire with a cross-sectional area of 0.15 cm^2. How much does the wire stretch? Neglect the mass of the wire. (0.44 cm for $Y = (2 \cdot 10^{11}\ \mathrm{N/m^2})$

EXAMPLE 8.10 A cube of Jello 6 cm on a side sits on your plate. You exert a horizontal force of 0.20 N on the top surface parallel to the surface and observe a sideways displacement of 5 mm. What is the shear modulus of the Jello? (670 N/m^2)

EXAMPLE 8.11 A marble column of cross-sectional area 1.2 m^2 supports a mass 25 000 kg. Young's Modulus of marble is $5 \cdot 10^{10}$ Pa . (a) What is the stress within the column? (b) What is the strain? ($2 \cdot 10^5$ Pa ; $4.1 \cdot 10^{-6}$)

9 FLUID MECHANICS

Fluids are liquids or gasses. Fluids can be compressible or incompressible. This is the big difference between liquids and gases, because ideal liquids are generally incompressible and without inner friction, while ideal gases are compressible and without inner friction. We can divide the fluid mechanics into hydrostatics and hydrodynamics. When talking about hydrostatics we can consider liquids to be ideal as inner frictions plays role only by liquid flow.

Fluids at rest – hydrostatics

Pressure p is defined as force per unit area, where the force F is understood to be acting perpendicular to the surface area A: $p = F/A$

The SI unit of pressure is N/m^2 which has the name Pascal (Pa); 1 Pa = 1 N/m^2 .

An experimental fact is that fluid exerts a pressure in all directions. Another important property of fluid at rest is that the force due to fluid pressure always acts perpendicularly to any surface which is in contact with it.

Hence the pressure p is defined as

$$p = F/A = \rho Agh/A = \rho gh$$

This equation tells us what the pressure is at depth h in the liquid due to the liquid itself. In an open container the pressure in depth h is:

$$p = p_0 + \rho gh$$

In this equation we have the pressure p_0 due to the atmosphere above the liquid surface plus the pressure of the liquid in depth h.

Pascal's law states that when there is an increase in pressure at any point in a confined fluid, there is an equal increase at every other point in the container.

Archimedes' principle: Any object, wholly or partially immersed in fluid, is buoyed up by force equal to the weight of the fluid displaced by the object.

Surface tension γ : The surface of liquid acts like it is under tension, and this tension, acting parallel to the surface, arises from the attractive force between the molecules.

This effect is caused by so called surface tension γ. It is defined as the force F per unit length that acts across any line in a surface;

$\gamma = F/l$ The unit of the surface tension is $[\gamma]$ = N / m

Fluids in motion - hydrodynamics

Fluids display properties such as: not resisting deformation, or resisting it only lightly (viscosity), and the ability to flow (also described as the ability to take on the shape of the container).This also means that all fluids have the property of fluidity.

The equation of continuity: When fluid is in motion, it must move in such a way that the mass is conserved. The mass flow rate is simply the rate at which mass flows past a given point, so it's the total mass flowing past divided by the time interval

$$\frac{\Delta m}{\Delta t} = \frac{\rho \Delta V}{\Delta t} = \frac{\rho A \Delta l}{\Delta t} = \rho A v \qquad \rho_1 v_1 A_1 = \rho_2 v_2 A_2$$

where ρ is the density, *v* is the velocity and *A* the cross sectional area.

Generally, the density stays constant and then it's simply the flow rate (Av) that is constant. This is a statement of the principle of mass conservation for a steady, one-dimensional flow, with one inlet and one outlet. This equation is called the continuity equation for steady one-dimensional flow. When there are many inlets and outlets, the net mass flow must be zero.

Bernoulli's equation: In steady flow of ideal fluid in gravitational field is the sum of kinetic and potential energy of unit volume and pressure along the line of current constant.

$$(1/2)\rho v^2 + \rho g h + p = const$$

In which the first member expresses the kinetic energy of unit volume, the second is potential energy of unit volume and p is pressure. This equation must have the same value at any point of the current tube.

EXAMPLE 9.1 Atmospheric pressure in various heights.

Solution

The equation $p = p_0 + \rho g h$ deals with incompressible fluids, the air is compressible. Let us imagine an element of air of area A and height dh with densityρ, so its weight is $\rho g A \mathrm{d}h$. The up-force on the element from below is pA and the down-force is $(p + \mathrm{d}p)A + \rho g A \mathrm{d}h$. At the hydrostatic balance:

$$pA - (p + \mathrm{d}p)A - \rho g A \mathrm{d}h = 0$$

$$\frac{\mathrm{d}p}{\mathrm{d}h} = -\rho g$$

According to ideal gas law $\rho = \frac{pM}{RT}$, where M = 0.02896 kg mol^{-1} is the middle molar mass of atmospheric gases, R = 8.314 J·K^{-1}·mol^{-1} is the universal gas constant and T the absolute temperature. So:

$$\frac{\mathrm{d}p}{\mathrm{d}h} = -p\frac{Mg}{RT} \qquad or \qquad \frac{\mathrm{d}p}{p} = -\frac{Mg}{RT}\mathrm{d}h$$

Integrating this equation among searching heights respectively pressures gives:

$$\int_{p(h_0)}^{p(h)} \frac{\mathrm{d}p}{p} = -\int_{h_0}^{h} \frac{Mg}{RT}\mathrm{d}h = -\frac{Mg}{RT}\int_{h_0}^{h}\mathrm{d}h$$

assuming *M, g,* and *T* are constant, hence atmosphere is isotherm. This simplyfying gives:

$$\ln\left(\frac{p(h)}{p(h_0)}\right) = -\frac{Mg}{RT}(h - h_0) = -\frac{Mg}{RT}\Delta h$$

$$\frac{p(h)}{p(h_0)} = e^{-\frac{Mg}{RT}\Delta h} \quad \text{or} \quad p(h) = p(h_0)e^{-\frac{Mg}{RT}\Delta h} = 1013{,}25\, e^{-\frac{\Delta h}{8432}} \text{ (hPa)}$$

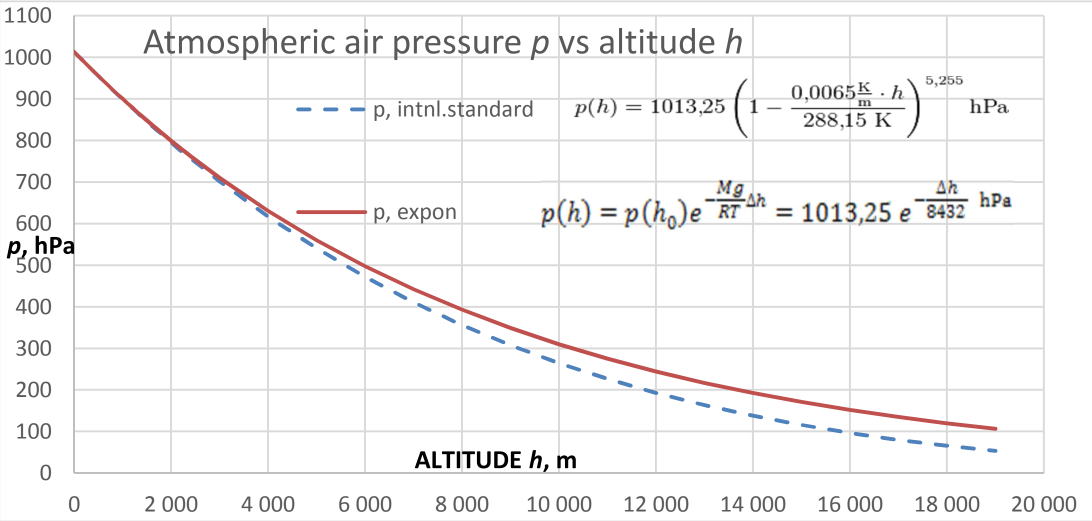


Fig. 9.1 Atmospheric air pressure vs altitude.

Figure 9.1 shows that even this simple approximation gives good results for small altitudes.

EXAMPLE 9.2 An iceberg floating in seawater is extremely dangerous because most of the ice is below the surface. This hidden ice can damage a ship that is still a considerable distance from the visible ice. What fraction of the iceberg lies below the water level?

Solution

The weight of the iceberg is $G_i = \varrho_i V_i g$, where $\varrho_i = 917\ \mathrm{kg\cdot m^{-3}}$ and V_i is the volume of the whole iceberg. The magnitude of the upward buoyant force equals the weight of the displaced water $F_B = \varrho_w V_w g$ where V_w, the volume of the displaced water, is equal to the volume of the ice beneath the water and $\varrho_w = 1030\ \mathrm{kg\cdot m^{-3}}$ is the density of the seawater. Because $\varrho_i V_i g = \varrho_w V_w g$, the fraction *f* of ice beneath the water`s surface is

$$f = \frac{V_w}{V_i} = \frac{\varrho_i}{\varrho_w} = \frac{917\ \mathrm{kg\cdot m^{-3}}}{1030\ \mathrm{kg\cdot m^{-3}}}$$

$$f = 0.89 \text{ or } 89\ \%.$$

EXAMPLE 9.3 A cube of wood having a side dimension a = 20.0 cm and a density ϱ_{cube}= 650 kg/m^3 floats on water, (a) What is the distance d from the horizontal top surface of the cube to the water level? (b) How much lead weight must be placed on top of the cube so that its top is just level with the water?

Solution

(a)

$$m_{cube} = V_{cube} \cdot \varrho_{cube} = a^3 \cdot \varrho_{cube} = 5.2 \text{ kg}$$

$$F_{buoy} = m_{cube} \cdot g$$

$$\varrho_{water} \cdot V_{displ} \cdot g = m_{cube} \cdot g$$

$$V_{displ} = \frac{m_{cube}}{\varrho_{water}} = a^2(a-d)$$

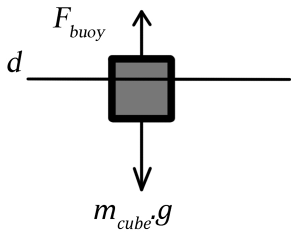


Fig.9.2 Cube on water

$$d = a - \frac{m_{cube}}{a^2 \varrho_{water}} = 0.2 \text{ m} - \frac{5.2 \text{ kg}}{0.04 \text{ m}^2 \cdot 1000 \text{ kg m}^{-3}} = 0.07 \text{ m}$$

(b) The mass $m_{to\ add}$ must be equal to the extra buoyant force we would get from submerging the part of the cube currently above water

$$m_{to\ add} \cdot g = \varrho_{water} \cdot V_{above\ water} \cdot g$$

$$m_{to\ add} = \varrho_{water} \cdot V_{above\ water} = 1000 \text{ kg m}^{-3} \cdot 0.07 \cdot 0.2 \cdot 0.2 \text{ m}^3 = 2.8 \text{ kg}$$

EXAMPLE 9.4 A large storage tank is filled to a height h_0. The tank is punctured at a height h above the bottom of the tank. Find an expression for (a) how far from the tank the exiting stream lands and (b) the maximum for it

(a) The Bernoulli`s principle for fluid at height h in the tank and after the hole

$$p_a + \rho g(h_0 - h) + \frac{\rho v_0^2}{2} = p_a + 0 + \frac{\rho v^2}{2}$$

$$v = \sqrt{2g(h_0 - h)}$$

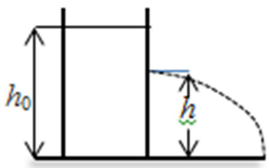


Fig.9.3 A tank filled to a height h_0

We neglected $v_0 \to 0$ since the tank diameter is much greater than the whole diameter. This is the Torricelli`s equation relating the speed of fluid flowing out of an opening to the height of fluid above the opening.

The fluid after the opening behaves as the horizontal projectile motion:

$$x = vt, \quad h = \frac{gt^2}{2}, \quad \text{or} \quad t = \sqrt{\frac{2h}{g}}$$

Using the Torricelli`s equation and the expression for *t* we receive:

$$x = vt = \sqrt{2g(h_0 - h)}\sqrt{\frac{2h}{g}} = 2\sqrt{(h_0 - h)h}$$

(b) An extreme of this function we find by derivation *x* according to *h:*

$$\frac{dx}{dh} = \frac{d}{dh}\left(2\sqrt{(h_0 - h)h}\right) = \frac{2(1/2)}{\sqrt{(h_0 - h)h}}(-h + h_0 - h) = \frac{h_0 - 2h}{\sqrt{(h_0 - h)h}}$$

So $h_0 - 2h = 0,$ or $h = h_0/2.$

The stream receives the maximal distance from the tank when the opening is in the middle of the fluid height.

EXAMPLE 9.5 In humans, blood flows from the heart into aorta, from which it passes into the major arteries. These branch into small arteries, which in turn branch into myriads of capillaries. The blood then returns through veins to the heart. The radius of aorta is about 1.2 cm, and the blood passing through it has a speed of about 40 cm/s. A typical capillary has a radius of about $4 \cdot 10^{-4}$ cm, and blood flows through it at a speed of about $5 \cdot 10^{-4}$ m/s. Estimate the number of capillaries in a body.

Solution

We assume the density of blood is constant. By equation of continuity, the volume flow rate in the aorta must equal the volume flow rate through all the capillaries. The total area of all capillaries is given by the area of one capillary multiplied by the total number *N* of capillaries. Let $A_a$be the area of the aorta and A_{Nc} the area of all *N* capillaries. Then $A_{Nc} = N\pi r_c^2$. From the equation of continuity we have

$$v_c A_{Nc} = v_a A_a \quad \text{or} \quad v_c N\pi r_c^2 = v_a \pi r_a^2$$

So

$$N = \frac{v_a}{v_c}\frac{r_a^2}{r_c^2} = \left(\frac{0.4 \text{ m/s}}{5 \cdot 10^{-4} \text{ m/s}}\right)\left(\frac{1{,}2 \cdot 10^{-2}\text{m}}{4 \cdot 10^{-6}\text{m}}\right)^2 \sim 7 \cdot 10^9$$

There are approximately 10 billion capillaries in the human body.

EXAMPLE 9.6 Water circulates throughout a house in a warm–water heating system. If the water is pumped at a speed of 0.5 m/s through a 4 cm diameter pipe in basement under a pressure of 300 kPa, what will be the flow speed and pressure in a 2.6 cm–diameter pipe on the second floor 5 m above? Assume the pipes do not divide into branches.

Solution

$$p_1 = 3 \cdot 10^5 \text{ Pa}, \qquad y_1 = 0, \qquad v_1 = 0.5 \text{ m/s}, \qquad y_2 = h = 5 \text{ m}$$

$$v_1 A_1 = v_2 A_2 \;\rightarrow\; v_2 = v_1 \frac{A_1}{A_2} = 0.5 \frac{\pi\, 0.02^2}{\pi\, 0.013^2} = 1.18 \text{ m/s}$$

$$p_1 + \rho g y_1 + \frac{\rho v_1^2}{2} = p_2 + \rho g y_2 + \frac{\rho v_2^2}{2}$$

$$p_2 = p_1 + \rho g y_1 + \frac{\rho v_1^2}{2} - \frac{\rho v_2^2}{2} - \rho g y_2 =$$

$$3 \cdot 10^5 + 0 + \frac{1000}{2}(0.5^2 - 1.18^2) - 1000 \cdot 9.8 \cdot 5 = 2.5 \cdot 10^5 \text{ Pa}$$

EXAMPLE 9.7 A piece of aluminum with mass 1.00 kg and density 2 700 kg/m^3 is suspended from a string and then completely immersed in a container of water .Calculate the tension in the string (a) before and (b) after the metal is immersed. The density of water is 1 000 kg/m^3. ((a) 9. 61 N, (b) 6.17 N)

EXAMPLE 9.8 Consider 2 identical pails of water filled to the brim. One pail contains only water, the other has a piece of wood floating on it. Which pail has the greater weight?

Solution

The buoyant force on the wood object equals the weight of the wood object (static equilibrium). Also the buoyant force is equal to the weight of the displaced water (Archimedes' principle). In other words, the weight of wood is the same as the weight of the spilled water. Therefore, putting the wood does not change the weight of the pail.

EXAMPLE 9.9 Water flows through a fire hose of diameter 6.33 cm at a rate of 0.012 m^3/s. The fire hose ends in a nozzle with an inner diameter of 2.20 cm. What is the speed at which the water exits the nozzle? (31.6 m/s)

EXAMPLE 9.10 A 70 kg ancient statue lies at the bottom of the sea. Its volume is $3 \cdot 10^4$ cm^3. How much force is needed to lift it? (392.4 N)

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