

Slovak University of Technology in Bratislava, Faculty of Civil Engineering, Department of Concrete Structures and Bridges

Jakub Dobrý

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Nonlinear analysis of slender reinforced concrete columns

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Submitter:	Ing. Jakub Dobrý Department of Concrete Structures and Bridges Faculty of Civil Engineering Slovak University of Technology in Bratislava Radlisnkého 11, 810 05 Bratislava
Supervisor:	prof. Ing. Vladimír Benko, PhD. Department of Concrete Structures and Bridges Faculty of Civil Engineering Slovak University of Technology in Bratislava Radlisnkého 11, 810 05 Bratislava
Readers:	prof. Ing. Jaroslav Halvoník, PhD. Department of Concrete Structures and Bridges Faculty of Civil Engineering Slovak University of Technology in Bratislava Radlisnkého 11, 810 05 Bratislava doc. Ing. Lukáš Vráblík, PhD. Faculty of Civil Engineering Czech Technical university in Prague Thakurova 7, 166 29 Prague
	doc. Ing. Peter Koteš, PhD. Department of Structures and Bridges Faculty of Civil Engineering University of Žilina Univerzitna 8215/1, 010 26 Žilina
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Prof. Ing. Stanislav Unčík, PhD. Dean of Faculty of Civil Engineering STU,

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Abstract

Columns have been a part of structures since the beginning of construction; they have retained their design and construction importance to the present. The advantages of using slenderer compressed elements are the consumption of less material and having more usable space in interiors. The continuous improvement of building materials and the use of hybrid structural elements has led to the downsizing of compressed structural elements. The smaller size of the columns leads to the higher slenderness of these structural members. Slenderer columns have lower resistance in bending and there are much more likely to lose stability. The subject of the dissertation thesis is a nonlinear analysis of the slender reinforced concrete columns and the loss of stability.

Nonlinear calculations can be considered as the most accurate calculation option for the load-bearing structural elements. This is the reason why is this thesis closely linked to practice, as in Eurocode 2 (1) in Chapter 5.8.6 of the European Concrete Design Standard there is the possibility of using the general nonlinear method in practice for compressed elements. In the design of slender structures, the influence of second-order theory is a very important part of the design. Loss of stability is another phenomenon this dissertation focuses on. The dissertation thesis presents a theoretical and experimental analysis of the slender columns, that fail due to loss of stability inside of their design interaction diagram. For this reason, the six specimens of slender columns were designed, fabricated and tested in the laboratories of Vienna University of Technology (TU Wien). All of the column specimens failed due to the loss of stability inside of their design interaction diagram which allows for the reliability analysis of the design method in Eurocode 2.

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1. Introduction

The topic of this dissertation thesis is a nonlinear analysis of the slender reinforced concrete columns and failure due to the loss of stability. Columns have been a part of structures since the beginning of construction; they have maintained their design and construction importance to the present. Slender structures are a very popular feature in modern architecture, and with the advancement of computer technology, it is much easier to design these elements. The advantages of using slenderer compressed elements are the consumption of less material and more usable space in interiors. The continuous improvement of building materials and usage of hybrid structural elements has led to the downsizing of compressed structural elements. Slenderness has a significant impact on the behaviour of columns under a compressive load, especially due to their lower bending stiffness, slender columns are likely to lose stability.

Nonlinear analysis can be considered as the most accurate calculation option for the load-bearing structural elements. With the advances in building modeling and calculations, there is also a great risk of confusion and inaccuracies that are directly related to their use. This is the reason why is this thesis closely linked to practice, as Eurocode 2 (1) in Chapter 5.8.6 of the European Concrete Design Standard allows the possibility of using the general nonlinear method in practice for compressed elements. Nonlinear calculations are a common part of the practice for calculating deformations of elements preferably exposed to bending such as beams and slabs. The calculations of compressed elements such as columns are much more advanced and place more demands on either the software or the designer

In the design of slender structures, the effect of the theory of second-order is a very important part of the design process. The nature of the theory of second-order is to increase the bending moment in the column due to its deformation. As the deformation increases, the eccentricity of the applied force increases, thereby increasing the bending moment of the element. Loss of stability is another of the phenomena this thesis focuses on. It is a form of brittle failure. Stability failure is characterized by no indications of upcoming failure as an excess deformation or the existence of cracks. The loss of stability is recognized in the design process as a failure inside the interaction diagram which represents the material capacity of the critical cross-section.

The dissertation thesis concentrates on the theoretical and the experimental analysis of the slender reinforced concrete columns. As a part of the experimental preparations, a column which design was based on numerous analyses, was tested in the laboratory. After the tests, the results were analysed and are presented in the thesis. Based on the material parameters, the nonlinear calculational model was created and used for the parametric study. As a part of experimental verification, the prediction of behaviour of tested specimens was performed by international experts. This thesis and research was performed for one reason, which is the analysis of the safety of the General nonlinear method.

2. The current state of knowledge

The combination of simultaneous effects of the axial force and the bending moment in two axes can be replaced by the effects of two eccentrically placed forces. The arm lengths of these forces depend on the size of the bending moment. Part of the design is also taking into account the buckling lengths relative to each axis. In addition to the eccentricity, which value depends on the applied bending moment, it is also necessary to consider the deformations based on the second-order theory. Due to the low tensile strength of the concrete, cracks in the column cross-section depend on the eccentricity of the applied load. In the case of a primarily compressed critical cross-section, five, four, or triangular distributions of the compressed concrete may occur.

2.1. The historical process of numerical analysis of concrete columns

The calculation basis of concentrically compressed elements were created by Leonard Euler. He assumed the sinusoidal deformation of the element along its length and the critical force (Euler force) calculated by 2.1.

$$N_{crit,m}^{II} = \frac{\pi^2 \cdot EI_z}{h_{ef,v}^2} \tag{2.1}$$

where *E* Young modulus

 I_z Moment of inertia

 $h_{ef,y}$ effective length related to the axis

Another pioneer of calculations of the compressed elements was Claude-Luis Navier, who recognized that, in addition to the loss of stability, critical section failure could also occur with respect to the theory of the second order. However, this phenomenon was generally known only on the basis of the experiments performed by prof. L. Telmajer (2). Another researcher who contributed to the development of the calculational techniques was Engesser (3) in 1889. He managed to calculate the resistance of the slender elements from the materials capable of transferring tension using a buckling instead of the Euler's stiffness. The assumed coefficient for the buckling was constructed from the moments of inertia of the compressed and stretched zones, as well as the modulus of elasticity dependent on the behaviour of the material at the relative deformation at the load limit.

In 1949 Haller (4) studied the load-bearing capacity of eccentrically loaded unreinforced masonry walls. The study was centered on any stress-strain relationship. He reduces the calculations of the members load-bearing capacity at the critical section to the equilibrium condition.

Kirtschig et al. in 1975 (2) used a parabolic stress-strain relationship of the Haller model and decided to neglect the tensile strength. With the determination of this boundary condition, the load-bearing capacity for the cross-sectional failure according to the theory of the second-order can be calculated as a mathematically closed relationship for the sections with cracks. A similar approach can be found in the work of Bazant et al. from 1991 (5) and 1997 (6), who also assumed the sinusoidal shape of the deformation along with the member. His calculations are based on the equal state of the force and the condition of relative deformations in the cross-section. This work group centered around prof. Bažant introduced the effects of material nonlinearity of concrete and steel, which lead to the extensive research of the slenderness ratio and its effects on the interaction diagram.

Christian Glock in 2001 presented for the first time the closed solutions for the system load capacity of slender and compact compression elements. This solution calculates with the linear-elastic material behaviour with and without consideration of the tensile strength (7). Another research group Kim et al. (8) (9) presented a numerical method that considered the effects of the material and the geometrical nonlinearities. The algorithm used in their model was confirmed by the extensive laboratory tests and studies. One of the researchers, who has been studying the loss of stability is prof. Bo Westerberg (10). Prof. Westerberg from Stockholm Technical University was one of the leaders of the comities that are in charge of the Eurocodes.

Very useful research in the area of experimental verification of slender columns was carried out by Foster and Attard (11) who focused on the effect of concrete strength on the ductility of columns. Another important source of information is the research of Claeson and Gylltoft (12), who analysed the effects of various slenderness, concrete strength, and initial eccentricity. 32 experimental columns from high strength and normal strength concrete were tested by Leite et al (13) who focused on unequal eccentricities. Broad research of 56 pinned high strength columns that were tested by Pallarés et al. (14), underlines the fact of the brittle failure of columns with higher slenderness and higher concrete compressive strength. The analytical study that focused on numerical analysis of various concrete slender columns was done by Júnior (15) who collected results from various researchers and analysed the effect of the reinforcement ratio, concrete strength, slenderness ratio, and dimension of crosssection and compared the results with Brazilian code. Another research performed on lightly reinforced columns was done by Porras (16) and discovered the brittle failure but not on the observed crack patterns. Fenollosa (17) observed the behaviour of the columns based on the slenderness, eccentricity and mainly focused on the differences between the normal and high strength concrete.

Bouchaboub (18) has focused his research on developing a nonlinear computational model for biaxial bending. Based on the results of his study that analyzed 15 columns with different slenderness and initial eccentricity he states that the in case of slender columns the stability failure is much more common than material failure.

The safety of nonlinear and probabilistic calculation has been questioned by multiple researchers due to the fact that the overall safety based on the safety factor was originally created for design principles that were used in past (method of allowed stresses) and not for the limit states that are used in Eurocode 2. Stability failure and nonlinear calculations are also methods that are not primarily based on the analysis of cross-section. As stated by Wolinski (19) and Holicky (20), there are more suitable safety methods for nonlinear calculations than Method based on the partial safety factors.

Several doctoral studies were focused on nonlinear analysis of columns, loss of stability, and slender reinforced concrete columns at the Slovak University of Technology in Bratislava (SUTBA), specifically at the Department of concrete structures and bridges. Most of these studies were supervised by prof. Vladimír Benko and prof. Ľudovít Fillo. All these studies had an impact on this thesis and formed a valuable foundation for the research.

2.2. Stability loss of slender reinforced concrete columns.

Stability loss of columns is tightly connected with their slenderness ratio that can be calculated by 2.9

$$\lambda = \frac{l_0}{i} \tag{2.2}$$

Where l_0 is an effective length of the column

i radius of gyration
$$i = \sqrt{I/A}$$

The effective length of columns depends on its boundary conditions

Slenderness has a significant impact on columns' behaviour under load, particularly due to the theory of the second order. As can be seen in Fig. 2-1 regardless of slenderness ratio, the bearing force of the columns always has an eccentricity e_0 . This eccentricity comes either from columns internal forces $e_1 = M/N$ or in case of column that is not affected by bending moment $e_i = l_0/400$ which is taking into account geometrical uncertainties. $e_0 = h/_{30} \ge 20mm$ is a sum of e_1 and e_i . The effect of this theory consists in the increase of the bending moment by increment caused by the deformation of the columns. This eccentricity is given by the Eurocode 2 (1) and it represents the geometrical uncertainties. The increment M_2 is determined by the eccentricity $e_2 = M_2/N$. Effects of the theory of the second order can by neglected if the increment of the bending moment is less than 10% of the bending moment from the theory of the first order.



Fig. 2-1: Effect of the slenderness on columns resistance

In cases of short or robust columns Fig. 2-1 a), deformations are so small that there is no need to consider them. Critical force in the cross-section is achieved by reaching the limit strength of structural materials. Failure is due to the compression of concrete or elongation of reinforcement.

In cases of regular columns, with an increase of normal force, the column deforms. With increasing deformation, the eccentricity of the second order e_2 increases. This increase can be seen in Fig. 2-1 b) and it is represented by the green (dashed) line.

The behaviour of the slender columns is also shown in Fig. 2-1 c). Due to the low bending stiffness with increasing normal force the deformation of the column rises rapidly up to the point when the level of force reaches the critical force N_B . This is the value of the normal force for stability failure. After the column loses its stability deformations rise rapidly without an additional increase of the normal force what leads to the increase of the bending moment up to the point of failure of the materials in the critical cross-section. As it was observed with experimental specimens, cross-section failure follows the stability failure in instance. The interaction M-N diagram represents the maximum material capacity of the critical cross-section, so if the column fails inside this diagram, the governing mode of failure becomes the loss of stability. The characteristics of this failure as the absence of cracks, low deformations and failure inside of design interaction diagram are not typical for columns but it can appear as mentioned in (16) (13).

2.3. Characteristics of a stability failure

In some cases of the columns with a high slenderness ratio, the failure of the columns can occur inside of their design interaction diagram. The loss of stability is very dangerous type of failure because it is a brittle failure that occurs without any warnings. Materials in the cross-section are not fully utilized, because, with this type of failure, the relative deformation of the compressed concrete is less than 3,5‰, usually around 1,5‰, which is significantly lower than the ultimate strain

3. Objectives of the dissertation thesis

Goals and objectives of the dissertation thesis with title: Non-linear analysis of slender reinforced concrete columns are:

- Numerical and experimental verification of assumptions that the slender reinforced concrete columns may fail by the loss of stability, inside of their design interaction diagram, with the maximum strain of compressed concrete in the critical cross-section under 1,2‰ and without the presence of cracks up to the point of failure.
- Prediction of the experimental columns behaviour by the experts from Europe. All of the predictions should be collected before the testing commencement.
- Assessment of the overall reliability of the columns designed by the general nonlinear method that failed due to the stability loss before the failure of theirs critical cross-section.
- Overall reliability of design methods in Eurocode 2 with the material model based on the results of the experiments that would be used for representing the most accurate behaviour of the tested columns in a variety of parametric studies.
- Comparison of the various approaches found in the codes and standards around the world.
- Sum up of the results of the performed experiments, parametric studies, and all of the analyses performed during the research and presenting the results of our obtained knowledge to the experts in charge of drafts of second generation of Eurocode 2 and general structural engineers.

4. Experimental verification

Experimental verification is a crucial part of the study. It serves to confirm the presented assumptions and theories. As already mentioned, this work is a sequel to the previous studies caried out at the Department of concrete structures and bridges. Experiments were performed in the laboratories of TU Wien in Vienna.

4.1. Pre experimental analysis

After the study of previous researches, the aim was placed on the nonlinear analysis. Numerous nonlinear numerical models of previously tested columns were created to find the best ways how to predict the behaviour of the tested columns. The approach by Bouchaboub (18) was an inspiration for the process of development a valuable computational model. After this initial research, the preparations for the experiment began. Laboratory of TU Wien had the possibility of testing the columns under controlled deformations, maximum force capacity of 17,5 MN, and a possibility to test the columns with the length of 5 meters. All of these parameters were not possible to reach in the Central laboratory of SUTBA (Slovak University of Technology in Bratislava).

The first step was to set a slenderness ratio, which had to be higher than λ =90. With the length of the columns 5 meters and reasonable height of cross-section 150mm. The next step was to find the width of the cross-section, reinforcement area, concrete strength class, and eccentricity of the load with the usage of nonlinear analysis in the software SOFiSTiK and Stab2NL. As a result of these efforts a cross-section that met the requirements was found. It is a cross-section marked S14 with concrete strength class C50/60 and eccentricity $e_i = 5mm$. The concrete strength class used for the selected cross-section was C50/60 and the reinforcement B 500B. Dimensions of the proposed cross-sections can be seen in Fig. 4-1 and Fig. 4-2,

The slenderness ratio of the columns was $\lambda = 115$ calculated by (4.3).

$$\lambda = \frac{l_0}{i} = 115 \tag{4.1}$$

Where $\ell_0 = 5.0$ m is an effective length of the column

i is the radius of gyration $i = \sqrt{I/A} = 0.0434 m$.



Fig. 4-1: Cross-sections of the tested columns





Fig. 4-2: Shape and reinforcement scheme of the tested columns

A column with a cross-section marked as S14, which is represented in Fig. 4-2 and Fig. 4-1 was analysed with the software SOFiSTiK and CUKON met the specification for the experimental verification. In addition, based on the nonlinear analysis, the column should not have any cracks prior to loss of stability. That underline the fact of the brittle failure. The total deformation in the mid of the column, which can be represented as additional eccentricity, should be greater than the minimum eccentricity set by the STN EN 1992-1-1 (1), which is 20mm.

ΤU

SvF

4.2. The fabrication process of the columns

The columns for the experimental verification were cast in the prefabrication facility of Strabag company in Sered', Slovakia. Followign samples were cast for testing: 6 concrete cubes, 10 prisms, and 13 cylinders made out of concrete and 6 samples of rebar were also collected. Columns were cast in a horizontal position to ensure the highest quality of the concrete pour. During the preparation of the fabrication, the smallest manufacturing imperfections were demanded. All of the columns were cast simultaneously from the same concrete mix see



Fig. 4-3.

Fig. 4-3: Fabrication of the experimental columns

The quality of the concrete pour in the formwork was ensured by vibrators. The crew in charge of the fabrication was experienced, the whole process went without any problems and every step was made under the author's supervision. Because of the experiences from the previous experiments, the columns were tested in two positions due to the casting in horizontal position.

4.3. Test rig for experiment

Experimental verification was performed in the laboratories of TU Wien in Vienna. The test rig was simple and previous experiments performed by prof. Benko in these laboratories were used as a stepping stone. To ensure correct measurement, several control measuring units were installed to control if the columns are not moving in any undesired directions. The aim it this experiment was to confirm the possibility of stability loss of member designed by method introduced in chapter 5.8.6 of the Eurocode 2 (1).



Fig. 4-4: Simplified representation of the test rig.



Fig. 4-5: Simplified representation of the eccentricity

A lot of effort was used to prepare this experiment and several extra steps were made to be sure about the results. Few extra measuring units were installed that could be used for cross-check of the results. In the addition to that, we used 3D scans to measure column deformations.

4.3.1. Boundary conditions and eccentricity of the force

Boundary conditions of the slender columns generally used in praxis for buildings can be hinges on both sides due to their low bending capabilities. Because this thesis intends to stay connected to praxis, the same conditions were used for the experiments. To ensure these conditions, elements from the high-strength steel were used on each side of the columns. These elements had a dual function and were also used to provide the initial eccentricity of 5mm. During the application of the steel elements for eccentricity, high accuracy was necessary because of a relatively small initial eccentricity. The placement of the column in the testing rig was done manually. The columns were hanged on the crane, and each side was manually placed into the position. The position of the column relative to the steel plates of the testing rig was in the offset of 5mm of the centre of plates, which were in the geometrical centre of the jacks on one side and in the centre of load cells on the other. This method of application was selected to avoid uncertainties. On each side of the testing rig two relatively thick steel plates were placed with a thickness of 60 and 80mm. These steel plates were also made out of high-strength steel to prevent deformations from Hertz stresses. Quality of used material was necessary to avoid any deformations of the member for eccentricity and steel plates so the boundary conditions remained unchanged.

4.4. Material tests

Material tests were performed on the 6 concrete cubes and the 13 concrete cylinders. In addition, 3 tests of the reinforcement were performed. All of the concrete material tests were carried out in the laboratories of TU Wien and the tests of the reinforcement in the Central laboratories of the Faculty of Civil Engineering of SUT in Bratislava.

4.4.1. Material tests of concrete

Demanded concrete strength class for the columns was C50/60 and it was the only demanded specification. All of the material samples were always stored next to the columns in stable interior conditions. The size of the cubes was 150/150/150mm and the size of the cylinders Ø150/300mm. Material tests were performed in the same week as experiments, approximately 4 months (prefabrication of the columns: 28th of November 2018, material tests: 27th of March 2019) after they were cast. The concrete cubes were used for the destructive tests of the compressive strength. Cubes were measured and weighed before the tests. The cylinders were used differently. 7 cylinders were used for the tensile strength. Tensile strength was examined by the splitting test.

With the remaining 6 cylinders, several tests were performed. Firstly Young's modulus of elasticity was measured with a non-destructive test prescribed by the Austrian standard ONR 23303:2010 (21). This kind of test is sensitive to the quality of the surfaces in contact with the jack, because of this, the surfaces were smoothened with a water-cooled grinder, and the dimensions and weight of every individual sample were measured. After the Young's modulus tests, destructive tests of compressive strength were performed. During these tests, the work law of material samples was also examined. The work law of the concrete is a crucial part of the material model for nonlinear calculations. Work law was examined by the 4 LVDT. Material parameters of used concrete are not in match with

parameters set by the code for concrete strength class C50/60. As can be seen in the following Table 1, the results of the tests are different than expected. Compressive strength is higher and young modulus much lower than expected. The concrete class that represents compressive strength could be categorized as C60/75 what is 2 strength classes higher than the concrete demanded. Based on Young's modulus the concrete can be categorized as C40/50 which is on the other hand 2 classes lower than demanded.

Date: 27.3.2019							
Maula	weigh	Length	diameter	Ec	F	fc	
Магк	(kg)	(mm)	(mm)	(GPa)	(kN)	(N/mm ²)	
1	12,332	294,8	149,6	36,420	1232,91	69,77	
2	12,533	297,18	150,4	36,887	1157,83	65,52	
3	12,659	298,42	150,3	34,212	1277,35	72,28	
4	12,459	296,6	149,81	34,177	1252,37	70,87	
5	12,597	297,4	149,9	35,569	1202,61	68,05	
6	12,482	296,14	150,15	37,272	1233,23	69,79	
Mean	12,510	296,757	150,027	35,756	1226,050	69,380	

Table 1: Results of cylinder compressive and young's modulus tests

4.4.2. Material tests of reinforcement

Material tests of reinforcement were performed in the Central laboratories of SUTBA on the 24th of July 2019. Tests have shown that the used reinforcement can be categorized as a B 500B.

4.5. Main experimental tests

The experimental testing was carried out in three days approximately 4 months after columns were casted. Each column was placed into position by a crane and manually secured in a precise position. All of the measuring units were installed afterward. As can be seen in *Fig. 4-6*, the columns were tested in a horizontal position. The column was loaded by force 100 kN. In this load step, chains hanged from a crane were loosened and the column deformed due to its self-weight. The bending moment from the self-weight was 4.2 kNm and the additional deformation from the self-weight approximately 5 mm. This is the reason for a sudden jump in the results. The effects of the self-weight are incorporated in the e_2 . The imperfections from fabrication were small and could neglected due to the precise fabrication and positioning of the column. Deformation from creep during the stage when

the columns were stored were also not present because the maximum compressive stress from self-weight was 1MPa and the reinforcement ratio over 3%.



Fig. 4-6: The column inside the testing ring

In each load step the level of force was kept for about 5 minutes. In every load step, the columns were examined for the presence of cracks. The cracks were not discovered in any of the tested columns until the point of the loss of stability. For clearer crack observation the middle part of columns was painted white with a thin layer of lime. The absence of cracks was expected based on the preceded nonlinear analysis. In the results presented in *Fig. 4-7 Fig. 4-8* and *Fig. 4-9*, an increase or decrease of the force, which was caused by loading by deformation can be seen.



Fig. 4-7: Graphical presentation of the experimental test results: N-M diagram

As can be seen in *Fig. 4-7 Fig. 4-8* and *Fig. 4-9*, the experimental results of almost all of the column specimens are similar. The results of one column is separated from the others, i.e., the column specimen marked S4, which was the first column that was tested in an inverted position (S4,S5,S6) in comparison to the previous ones (S1,S2,S3). The dashed curves in the presented figures represent the experimental behaviour of all column specimen. Three interaction diagrams (ID) are displayed in *Fig. 4-7*; they represent the design, characteristic, and mean values of material properties of the critical cross-section. The actual concrete parameters were used for the ID calculation and were obtained by material tests. All of the columns reached their peak resistance (the points placed on the curves) inside of the design interaction diagram of the critical cross-section. In the presented figures, only 5 points can be seen. Two of the tested columns failed very similarly to each other and the dots overlay. This type of failure is characterized as the loss of stability. The S4, S5, and S6 columns were tested in an inverted position in comparison to the S1, S2, and S3 columns due to the fabrication process.



Fig. 4-8: Graphical presentation of the experimental test results of the N- e2 diagram



Fig. 4-9: Graphical presentation of the experimental test results N-ε diagram

In *Fig. 4-9*, the *N*- ε relation is displayed. The dashed curves represent the relative deformation of the top surface, and the dashed-dot curves represent the bottom surface from the LVDT measuring units. As can be seen, the concrete failed in compression with the average strain around 1.12 ‰. All of the columns tested, did not have any cracks in the last load step prior to the loss of stability. The cracks were monitored during the whole time of the experiments by a camera. The absence of cracks is additional evidence that the loss of stability has a brittle mode of failure and therefore the design should be taken care of accordingly.

Increased concrete strength affected the volume of the interaction diagram to rise but in the case of stability failure increase of concrete strength does not have a significant role because of the poor utilization of material as can be seen in Fig. 4-10. Poor utilization is caused by the low relative deformation of compressed concrete. The lower E-modulus of concrete on the other side had a tremendous impact on the results.



Fig. 4-10: Material work law of concrete - Utilization of material



Fig. 4-11: Material work law of reinforcement - Utilization of material

The level of material stress in the reinforcement was also low due to the low relative deformation of the critical cross-section. Curves in Fig. 4-11 represent curves from material tests of reinforcement. Relative deformation of reinforcement was around 1‰ and the approximate stress can be seen in the figure above. Khalil et al. [22], tested columns with similar slenderness, and the findings of his research are very similar to those presented in this thesis. Cracks were not present until the stability failure and the strain of the compressed concrete was very similar to the presented values of this series of tests.

4.6. Prediction of experimental behaviour

The recent development in software and the accessibility of necessary hardware has led to the increased usage of nonlinear design methods for the design of new concrete structures. The initial intent of the presented research was to point out the risks of nonlinear analyses in the case of compressed structural members. The results of nonlinear analyses are sensitive on a variety of factors. One of them is the so-called effect of the Black Box. A large number of engineers in praxis use software they do not completely understand, and a great possibility of design failure occurs as a result. Because of these risks, the experts from around Europe were asked, for their prediction of behaviour of the experimentally tested columns. The predictions were anonymous to prevent concerns of authors about their results. Every expert was provided with information about the details of the testing rig (Fig. 5), strength of materials, initial eccentricity, and the type of loading. All of the experts that entered this prediction have a large amount of experience with nonlinear calculations. They were asked to provide the values of the maximum normal force and the additional eccentricity e2 relevant to the maximum normal force. The stress-strain diagram was not prescribed and the experts were not asked to provide which stress-strain diagram was used for their caulculation. The results were collected before the experimental testing began and authors were not approached to modify their results at any time. Results of their predictions can be found in Fig. 4-12. It is important to state that the columns concrete was supposed to be from strength class C50/60 f_{cm} =58MPa) while the strength from the material tests was $f_{cm}(t)$ =69.38 MPa. This value represents concrete strength class C60/75 what is 2 classes higher in terms of strength than the concrete required.



Fig. 4-12: Predicted N-M diagrams of the experiment by each expert

In Fig. 4-12 the N-M behaviour of experiments and the experimental predictions are represented. Thin black lines represent the predictions from authors, thick black dashed lines represent the prediction with the highest and lowest capacity. The Grey area, therefore, represents the range of the predictions. Red dot-dashed curves represent the strongest and the weakest experimental specimen, and the red area represents the range of experimental results. Three interaction diagrams (ID) are displayed in the Fig. 4-12 which represents the design, characteristic, and mean material properties of the columns' critical cross-section. The considered concrete in these IDs is the demanded concrete (C50/60). As can be seen, the results are very different. The difference between the maximum and minimum prediction is 47% and the difference between the maximum prediction and weakest column is 45% in the value of normal force. Most of the calculated results by experts predicted, that the tested columns will collapse inside of the design interaction diagram. All the experimental specimens also failed inside of the design interaction diagram, which is the characteristic of the loss of stability. This type of failure is brittle and therefore it is very dangerous for actual structures. Some of the experts did not consider the self-weight of the columns, what can be seen in the results but for the authenticity of this prediction, they were not corrected. The calculation process of experts was also not analysed.

Nonlinear calculations are currently very popular for the optimization of the design. Based on the foundations of (22), the most relevant tool for optimization of the design process is the suitable structural concept of the structure. The nonlinear calculations could be a helpful tool because they can realistically represent the behaviour of the structures but as stated before, they require a lot of experience.

4.7. Post experimental numerical analysis

As a part of post-experimental research numerical linear and nonlinear analyses were performed with software SOFiSTiK and MATLAB. SOFisTiK is also mentioned in the previous part of the research as it was used for pre-experimental analysis. In MATLAB, scripts from the software FedeasLab (23) were used for nonlinear calculations. Scripts for the linear calculations were developed of modified by the author of the presented thesis.

4.7.1. Computational nonlinear model – FedeaSlab

FedeaSlab was used for the nonlinear finite element analysis of the beam model. This model has the capability of representing confined concrete but confinement was not used in the analyses. Elements in the cross-sections were segregated only in one dimension since the

experimental columns were exposed only to the bending in one axis. Elements had a prescribed corotational geometry and Gauss-Lobatto element integration was used. Solution engine was created by prof. Filippou at University of California in Berkeley. During the cooperation with the University of California, the solution process and material input strategy were modified for concrete elements. This software was chosen for the detailed analysis to prevent the effect of the black box. Results of FedeaSlab are presented in *Fig. 4-13*. The input parameters were not manipulated to fit the experimental results and as can be seen, the calculational behaviour is very similar to experiments. This material and computational model was further used for the parametric study presented in chapter 4.8.1. This calculational model consisted of 5 beam elements that were segregated into 5 finite elements each. The solution engine from FedeaSlab was based on the Timestep analysis.



Fig. 4-13: Comparison of a numerical model with experiments – N-M diagram

4.8. Safety of General nonlinear method

The motivation for the experimental work are doubts concerning of reliability of the General nonlinear method in Eurocode 2 (1) in the case of design compressed members when the loss of stability precedes the material failure of the critical cross-section. Generally, according to Eurocodes the global reliability of the design is ensured by the partial factors for loads and partial factors for the material strength. As the simplification, a global factor of reliability can be empirically express as (4.5)

$$\gamma_0 = \gamma_M * \gamma_F \tag{4.2}$$

By the use of the general method according to (1) chapter 5.8.6 in cases where a column subjected to axial force and loses stability, the design strengths of the concrete and reinforcement, are not always fully utilized, because of the low relative strain of the cross-section. It means that the partial factor for material γ_M is not fully included in the calculation of global reliability.



Fig. 4-14: Graphic representation of the safety in the case of the stability loss

In *Fig. 4-14* are three results of the General nonlinear method, where mean values of the actual material properties were used (red line), 5 % fractile - characteristic values (red dashed line), and finally results obtained with design material properties for concrete and reinforcement (black line). Following partial factors were used for design values: $\gamma_{\rm C} = 1.50$, $\gamma_{\rm CE} = 1.20$ for concrete and $\gamma_{\rm S} = 1.15$ for reinforcement. Interaction diagrams presented in this figure were also calculated with the same mean, characteristic, and design values of material properties. It is important to state that the red curve was obtained with a numerical and geometrical model that was calibrated by the results of experiments in chapter 4.5. Then the model was used for the calculation of the red-dashed and black lines, where only material properties were replaced by characteristic and design values. The calculations were carried out in the software FedeasLab (24).

Based on the analyses performed and presented in *Fig. 4-14*, the partial factor for the loss of stability can be expressed by the ratio $\gamma_{\rm B} = N_{\rm B,k}/N_{\rm B,d}$ and reached a value of 1.125. Buckling force $N_{\rm B,m}$ was assessed with actual material properties, $N_{\rm B,k}$ with characteristic,

and $N_{B,d}$ with the design values. All of the results were calculated with the General nonlinear method. It is necessary to state at this point that the loss of stability is a brittle failure and a safety margin of 0.125 expressed by $\gamma_B = 1.125$ is quite low for such a mode of failure.

As it is stated in the research of Wolinski (19), the safety factors for nonlinear calculations used by the Eurocode may not represent the real behaviour of structures and may impact the change in different failure type if they are used for the analysis of the whole structure and not just the specific structural member. The safety of the design is also sensitive to the type of failure and safety factors may not be the best solution for ensuring the safety of design for a structural member that is designed by nonlinear methods and fail due to the loss of stability.

4.8.1. Parametric study

The parametric study presented in this chapter was performed with software FedeaSlab with a material model calibrated by experiments. The columns in the laboratory were tested in a horizontal position but the calculated column behaviour presented in this part of the study considers columns in the vertical position what reflects the general column position. The study shows the development of safety factor with increased slenderness ratio and different values of initial eccentricity. Columns tested in a laboratory had the slenderness ratio λ =115 and this study also presents results of columns with slenderness ratio λ =130, λ =150, λ =175, λ =200.

Initial eccentricity used in experiments was $e_i = 5mm$ and the parametric study also investigates the increased value of $e_i = 10mm$ and $e_i = 20mm$. In the upcoming figures, three interaction diagrams calculated with mean, characteristic, and design material parameters are presented. Mean material parameters are from experiments. Characteristic value was calculated as 5% fractile of material tests and design value is calculated from characteristic value as prescribed in EC2 (1). These values were also used for the calculation of column behaviour. The calculation methods used in this parametric study are all three calculation methods from EC2. Namely the General nonlinear method (GNLM), the method of Nominal curvature, and the method of Nominal stiffness. Value of maximum normal force for Nominal curvature method and Nominal stiffness method is the point where the calculated response of these methods presented as curves intersect with design interaction diagram. In the tables below, the numerical comparison of results from three EC2 methods to the referent value of normal force calculated using GNLM with characteristic material parameters ($N_{Rk,gnlm}/N_{Rd,EC2}$) are presented. This comparison can be explained as a safety factor for the calculation method.

 Table 2: Development of safety factor for the General nonlinear method with various slenderness and initial eccentricity.

General nonlinear method - Design value						
Ratio N _{Rk,gnlm} /N _{Rd,EC2}						
-	λ=115	λ=130	λ=150	λ=175	λ=200	
e0=5mm	1,14	1,13	1,12	1,11	1,11	
e0=10mm	1,14	1,13	1,12	1,12	1,12	
e0=20mm	1,13	1,13	1,13	1,13	1,13	

The development of the safety factor for the general nonlinear method is relatively stable with a value around 1,12. As is mentioned at the beginning of chapter 4.8, the value of 1,12 is very low and does not leave much space for errors that can easily occur in practice. The value is so stable due to the calculation process since the reference value was calculated by the general nonlinear method with characteristic material properties.

 Table 3: Development of safety factor for the Nominal stiffness method with various slenderness and initial eccentricity.

Nominal Stiffness method						
Ratio N _{Rk,gnlm} /N _{Rd,EC2}						
-	λ=115	λ=130	λ=150	λ=175	λ=200	
e0=5mm	3,12	3,39	3,68	3,87	4,05	
e0=10mm	2,69	2,93	3,18	3,42	3,66	
e0=20mm	2,14	2,26	2,45	2,68	2,97	

The safety factor for the Nominal stiffness method decreased with increased initial eccentricity but increased with slenderness ratio. This effect is connected with increased deformation and bending moment which is more present with a higher slenderness ratio.

Nominal Curvature method						
Ratio N _{Rk,gnlm} /N _{Rd,EC2}						
-	λ=115	λ=130	λ=150	λ=175	λ=200	
e0=5mm	1,90	2,05	2,23	2,42	2,53	
e0=10mm	1,68	1,78	1,93	2,14	2,23	
e0=20mm	1,32	1,41	1,54	1,70	1,81	

 Table 4: Development of safety factor for the Nominal curvature method with various

 slenderness and initial eccentricity

Based on the presented research overall reliability of the design of slender structures is relatively low not only for the general nonlinear method but also for the Nominal curvature method. Nominal curvature method is simplified method and therefore the safety margin of 0.32 is quite low in comparison to the other simplified method of Nominal stiffness with margin 1.14 for the same slenderness ratio and initial eccentricity.

5. Conclusions

The presented dissertation thesis is focused on the safety the nonlinear analysis of slender reinforced concrete columns, because Eurocode 2 (1) in chapter 5.8.6 allows the use of the General nonlinear method in practice also for compressed elements. The referent columns specimen that was used for theoretical an experimental analysis is a result of extensive numerical analyses and research of previously tested columns. A reference column specimen was designed by the author and manufactured in the prefabrication facility. Six experimental columns were tested in the laboratories of the Technical University in Vienna along with the large number of material samples. The initial eccentricity of the force was set to 5mm and columns were tested in the horizontal position. All of the tested columns failed inside of their interaction diagram without the presence of cracks prior to the point of loss of stability, and with the maximum strain of the compressed concrete approx. 1.12‰, which are signs of a loss of stability.

Prediction of experimental specimen behaviour was also carried out before the start of experiments. The difference between the strongest and weakest prediction is 47% and between the strongest prediction and weakest experimental specimen in case of the normal force is 45%. The findings of this prediction only underline the risks connected with using the General nonlinear method. For complete determination of the safety of the GNLM, the β index should be observed. This thesis does not contain the analysis of the β index (reliability index) but focuses on the safety factor of the general nonlinear method. This safety factor is explained in the dedicated chapter and it is around 1.125 for experimentally tested columns.

The last part of the presented research is deals with the parametric study of experimental columns. Based on the presented research, the safety factor of the general nonlinear method is almost the same (1.11-1.13). The safety factor of the Nominal stiffness method and Nominal curvature method increases with the slenderness ratio and decreases with increasing initial eccentricity.

The analysis proves that the safety factor for the GNLM is very low even for the well-calibrated numerical model. For the practical use of this method by the structural engineers, the safety factor should be increased because as the prediction of experimental behaviour shown, even the well-experienced experts can make mistakes that could lead to underestimation or worse overestimation of the capacity of slender compressed elements.

5.1. Recommendations for general praxis

The cornerstone of the presented research is based on the possible unsafe design of slender reinforced concrete columns designed by the general nonlinear method. Nonlinear calculations of compressed elements are mostly used in praxis in two cases. The first one is connected to the economic aspect of real estate development and lowering the volumes of concrete and reinforcement. As was examined by Dobrý et al. (22) the correct and the most effective tool for optimization is the right structural concept. The second reason mostly results from unsafe design and nonlinear calculations are the tool for finding the reserves in the design. This usage should be left for the structural engineers with years of experience with nonlinear calculations and experimental verifications. If it is left for use by an inexperienced engineer, it can lead to even more unsafe design.

The possibility of unsafe design by the general nonlinear method was proven in the presented research. Due to the presented facts, the author would advise the responsible comities for the second generation of Eurocodes to remove the general nonlinear method for compressed elements from the actual standard to annexes. In addition to that, another safety factor should be added into this method. The safety factor for the loss of stability would increase the safety and reduce the possibility of design failure. The mentioned safety factor

is already placed in the Austrian and Slovak annex of Eurocode 2 (25). The general nonlinear method should also be used only for the analysis of existing structures and not for the design of new structures.

5.2. Recommendations for further research

The creation of several parametric studies focused on columns with slenderness ratios lower than λ =115, different reinforcement ratios, cross-sections, and concrete strengths. Results of these studies should be compared to the results of other standards as ACI 318-19 (26), with a focus on all of the design methods.

Further research in the field of slender columns should be also focused on the probabilistic analysis of tested specimens. Reliability index would be an important step toward the marking the general nonlinear method safe or unsafe.

Drafting of publication that would contain all the necessary information for the safe design of slender structural elements. This publication should contain comparisons of design methods with performed experiments, theoretical background, probabilistic analysis, and economic analysis so it could be used for the education of ordinary structural engineers.

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