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Report of dissertation thesis

MESHLESS FORMULATION FOR SMART STRUCTURES

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Abstract

In this dissertation thesis, a meshless method based on the local Petrov-Galerkin (MLPG) approach is proposed, to solve static and dynamic problems for smart structures that consist of magnetoelectroelastic (MEE) material. The local weak formulation is employed on circular subdomains. Subdomains surrounding nodes are spread over the analyzed domain. The moving least-squares (MLS) method is adopted for the approximation of the physical quantities. Mesh of finite elements is thus not needed for the analysis.

An enhancement of the magnetoelectric coefficient is analyzed in two-layered composite consisting of two dissimilar piezoelectric and piezomagnetic materials. The magnetoelectric coefficient is vanishing in pure piezoelectric as well as in pure piezomagnetic constituents. It is shown that the electric potential in the piezoelectric layer is induced by the magnetic potential in the piezomagnetic layer. Functionally graded material (FGM) properties are considered too. Many magnetoelectroelastic components are intended to act as an actuators or sensors, which have plate-like shapes. Thus MLPG method is also proposed for plate bending analysis with material having functionally graded magnetoelectroelastic properties. Axial symmetry of geometry and boundary conditions for a circular plate reduces the original 3D boundary value problem into a 2D problem in cross section of the plate. Large deflection of square plate is also analyzed. The von Karman plate theory of large deformations is applied to express the strains. Bending moments and shear forces are considered by the Reissner–Mindlin theory, and the original three-dimensional (3D) thick plate problem is reduced to a two-dimensional (2D) one. The dissertation thesis also discusses the crack analysis with a central crack on an interface between elastic and magnetoelectroelastic solids. The crack opening displacements, electric potential and intensity factors are investigated. The intensity factors are computed from the generalized crack-opening-displacements.

Numerical results presented in this thesis have shown excellent properties of the meshless MLPG method for the analysis of engineering structures with MEE material properties.

Keywords: Meshless local Petrov-Galerkin method (MLPG); moving least-squares approximation (MLS); magnetoelectric effect; magnetoelektroelastic material; functionally graded material; von Karman plate theory; intensity factors

Abstrakt

V tejto dizertačnej práci je pre riešenie statických a dynamických úloh inteligentných konštrukcií pozostávajúce s magnetoelektroelastických materiálov (MEE) uvažovaná bezprvková metóda založená na lokálnom Petrov-Galerkinovom prístupe (MLPG). Slabá lokálna forma riadiachich rovníc je použitá na malých kruhových oblastiach. Tieto podoblasti obklopujúce výpočtové body sú rozmiestnené po analyzovanej oblasti. Pre aproximáciu neznámych veličín je použitá metóda pohyblivých najmenších štvorcov. Použitie konečných prvkov nie je teda vôbec potrebné.

V práci je skúmané zlepšenie magnetoelektrického koeficientu v dvojvrstvovom kompozite pozostávajúcom z piezoelektrického a piezomagnetického materiálu. Elektromagnetický koeficient je nulový v obidvoch materiáloch, ale je ukázané, že elektrický potenciál je indukovaný v piezoelektrickej vrstve prostredníctvom magnetického potenciálu aplikovaného v piezomagnetickej vrstve. Taktiež sú uvažované aj funkcionálne gradované materiálové parametre. Množstvo magnetoelektroelastických materiálov sa používa ako senzory alebo aktuátory, ktoré majú často tvar kruhových dosiek. Preto je MLPG metóda uvažovaná pre analýzu ohybu MEE dosiek. Materiálové parametre sú uvažované ako funkcionálne gradované. Z dôvodu osovej symetrie je postačujúce pre kruhové dosky analyzovať len prierez dosky, čím sa 3D úloha redukuje na 2D. Veľké priehyby štvorcových dosiek sú tiež vyšetrované. Pre vyjadrenie pomerných deformácii je použitá Von Karmanova teória dosiek veľkých deformácii. Pre ohybové momenty a šmykové sily je uvažovaná Reissner-Mindlinova teória, čím sa 3D problém tenkých dosiek redukuje na 2D úlohu v prierezovej rovine dosky. Dizertačná práca sa tiež venuje centrálnej trhline na rozhraní magnetoelektroelastickej a elastickej vrstvy. Vyšetrované sú otvorenia trhliny, elektrické a magnetické potenciály a intenzitné koeficienty, ktoré sú počítané z hodnôt otvorenia trhliny.

Numerické výsledky predstavené v tejto dizertačnej práci ukazujú výhody použitia bezprvkovej MLPG metódy pre riešenie inžinierskych konštrukcií s MEE materiálovými vlastnosťami.

Keywords: bezprvková lokálna Petrov-Galerkinova (MLPG) metóda; metóda pohyblivých najmenších štvorcov (MLS); magnetoelektrický efekt; magnetoelektroelastický materiál; funkcionálne gradientné materiály; von Karmanova teória dosiek; intenzitné koeficienty

Contents

Introduction

Analysis of smart structures is currently in the scope of many engineers and researchers. The solution and complex engineering design problems of smart materials embedded in intelligent structures requires having an advanced computational tool in order to obtain results effectively and with required accuracy. It is necessary to develop advanced computational methods and modelling approaches taking into account specific properties of smart materials.

A wide range of engineering applications of smart materials have been found recently. Improvements of the performance of engineering structures lead also to developments and applications of new structural materials. Smart materials are new class of multifunctional materials that have one or more properties that can be controlled or changed by action of external effects in predetermined way, so they act as energy transducers and can be applied as sensors and actuators. Special categories of smart materials are piezoelectric, electrostrictive and magnetostrictive materials. They have the capability to serve as both sensors and actuators. By putting together piezoelectric and magnetostrictive materials, we are able to create a new product, magnetoelectroelastic (MEE) material. In MEE materials, the magneto-electric forces give raise to strains that can reduce the effects of the applied mechanical load. That is why MEE materials are solely investigated in the thesis.

Up to date multilayered composites are mostly analyzed. Therefore, it could be interesting to investigate also composites, where the volume fraction of constituents is continuously varying in a predominant direction. This type of property is called as functionally graded property. The thesis keeps focusing on developing advanced numerical method for anisotropic functional graded magnetoelectroelastic (MEE) solids as well as layered composites. The solution of general boundary value problems for such kind of solids requires advanced numerical methods due to the high mathematical complexity.

In recent years, meshless formulations are becoming popular due to their high adaptability and low costs to prepare input and output data in numerical analyses. The term “meshless” or “meshfree” stems from the ability of an approximation or interpolation scheme to be constructed entirely from a set of nodes without the need to connect them into elements. Meshless methods for solving partial differential equations (PDE) in physics and engineering sciences are a powerful new alternative to the traditional mesh-based techniques as finite element method (FEM) or boundary element method (BEM). Focusing only on nodes or points instead of elements, gives certain advantages to meshless approaches.

1. Outline and main contribution of the thesis

The aims of the thesis are:

- to provide a brief introduction to application of various smart materials applied in smart structures
- to focus on more detail description of magnetoelectroelastic materials
- to introduce meshless solution approach for the boundary value problems
- to describe in detail the MLPG method, specification of local integral equations and moving least squares (MLS) approximation technique
- to derive local integral equations for magnetoelectroelastic solid
- to extend the formulation for layered composite
- to analyze the enhancement of the magnetoelectric coefficient in layered or funcionally graded multiferroics
- to derive local integral equations for axisymmetric circular plates made of funcionally graded MEE material
- to derive local integral equations for nonlinear large deflection of rectangular MEE thick plates under a large deformation described by the Reissner– Mindlin theory
- to present state-of-the-art in the field of crack anlyses in MEE material
- to analyze interface crack between MEE and elastic layers

2. Magnetoelectroelastic (MEE) material

In the magnetoelectroelastic materials, we can observe a magnetoelectric (ME) effect. The ME effect was discovered in 1894 by Curie. It is defined as the ratio between the magnetic (electrical) field output over the electrical (magnetic) input. The coupling can be provided by materials in single phase or as composites. The single phases materials are represented by multiferroic magnetoelectric materials, in which both ferromagnetic and ferroelectric components are in the same phase, as well as magnetically ordered ferroelectric materials [1]. There are very few 'multiferroic' materials in nature or ones that have been synthesized in the laboratory that exhibit a stable and switchable both electrical polarization and magnetization [2]. From earlier investigations it is well known that some composite materials can provide superior properties compared to their virgin monolithic constituent materials. The ME composites have large magnitudes of the ME voltage coefficient. They are created from individual piezomagnetic and piezoelectric phases or individual magnetostrictive and piezoelectric phases [3]. In MEE material, the ME effect is a product of piezomagnetism or magnetostriction, in which a mechanical deformation is caused by a magnetic field, and piezoelectricity, in which an electric potential is induced by the mechanical deformation [4]. In turn, an applied electric field induces a magnetization via the mechanical coupling between the constituents. In contrast to the intrinsic ME effect of single phase crystalline samples the composite ME effect manifests predominantly as nonlinear in the applied fields.

Because of the complex nature of the ME interaction between the constituents the relation between the applied magnetic field and the voltage induced in the detection circuit is not simply linear as in the case of single-phase compounds. Due to the hysteretic nature of the ME effect, the composites may find applications in memory devices. The linear ME effect has a positive or a negative sign, depending on the annealing conditions (parallel or antiparallel magnetic and electric fields). In binary data storage devices the ME material can thus store information in two different states distinguished by the sign of the ME response. Such a memory will be an effective ‘read only’ memory, since the reading can be done at very high frequencies. Data writing is more difficult because it involves temperature annealing in magnetic and electric fields or the use of very high writing fields [3]. Further applications include magnetic field sensors and magnetically controlled optoelectronic devices. The transduction properties of the ME effect can also be employed in ME recording heads and electromagnetic pick-ups [5]. Historical perspective, status and future of multiferroic magnetoelectric composites are given in a review paper [6].

Constitutive equations of the coupling of the mechanical, electrical and magnetic fields in magnetoelectroelastic solids [4] are given as

$$\sigma_{ij}(\mathbf{x},\tau) = c_{ijkl}(\mathbf{x})\varepsilon_{kl}(\mathbf{x},\tau) - e_{kij}(\mathbf{x})E_k(\mathbf{x},\tau) - d_{kij}(\mathbf{x})H_k(\mathbf{x},\tau) \tag{2.1}$$

$$D_j(\mathbf{x},\tau) = e_{jkl}(\mathbf{x})\varepsilon_{kl}(\mathbf{x},\tau) + h_{jk}(\mathbf{x})E_k(\mathbf{x},\tau) + \alpha_{jk}(\mathbf{x})H_k(\mathbf{x},\tau) \tag{2.2}$$

$$B_j(\mathbf{x},\tau) = d_{jkl}(\mathbf{x})\varepsilon_{kl}(\mathbf{x},\tau) + \alpha_{kj}(\mathbf{x})E_k(\mathbf{x},\tau) + \gamma_{jk}(\mathbf{x})H_k(\mathbf{x},\tau) \tag{2.3}$$

where σ_{ij}, D_i, B_i represent the stress tensor, the electric displacements, and the magnetic inductions, respectively. Material parameters are the elastic coefficients c_{ijkl}, dielectric permittivities h_{jk} and magnetic permeabilities γ_{jk}. The coefficients for the piezoelectric, piezomagnetic, and magnetoelectric coupling are denoted by e_{kij}, d_{kij} and α_{jk}, respectively. Symbol τ is used for time. The strain tensor ε_{ij}, electric field vector E_i and magnetic intensity vector H_i are expressed through independent variables - displacements u_i, electrical potential ψ and magnetic potential μ.

Governing equations for magnetoelectroelastic body under dynamic loading conditions are given by the balance of momentum and the scalar Maxwell's equations as

$$\sigma_{ij,j}(\mathbf{x},\tau) + X_i(\mathbf{x},\tau) = \rho \ddot{u}_i(\mathbf{x},\tau) \tag{2.4}$$

$$D_{j,j}(\mathbf{x},\tau) - \Pi(\mathbf{x},\tau) = 0 \tag{2.5}$$

$$B_{j,j}(\mathbf{x},\tau) = 0 \tag{2.6}$$

where $\ddot{u}_i$, ρ, X_i and Π represent the acceleration of the displacement, the mass density, the body force vector and the volume density of free charges, respectively. In case of static loading conditions, right-hand side of equation (2.4) is equal zero.

3. Meshless MLPG method

In many cases there is a need to solve complex geometrical domains, and quite often, nonlinear problems of engineering processes. Exact mathematical solutions are available for the simplest geometrical domains and mostly for linear problems. Computer based modelling and simulation techniques were introduced in parallel with development of computer technology. There is a great deal of commonality in the differential equations governing the engineering process and so we are able to make common computational tools for computer modelling and simulation.

The finite element method (FEM) is well established and often used for the solution of general boundary value problems. The FEM in general would suffer from drawbacks such as locking in bending, demanding complicated mesh or re-meshing in problems of large deformations and C^0 continuity of approximated fields in analyses of functionally graded material, which means that secondary fields such as stresses lack the continuity across the element interfaces. Another well established method to the FEM is the boundary element method (BEM), which is effective and convenient numerical tool. Unfortunately, the drawback of BEM is that the fundamental solution is not available for functionally graded magnetoelectroelastic solids.

Meshless methods are alternative computational techniques. Several meshless methods can be derived either from a weak-form formulation on the global domain or a set of local subdomains. In the global formulation, background cells are required for the integration of the weak-form. In the methods based on the local weak-form formulation no background cells are required and therefore they are often referred to as truly meshless methods. Meshless method used in this thesis is the meshless local Petrov-Galerkin (MLPG) method. This local formulation is using nodal points, which are spread on the analyzed domain and each of them is surrounded by a small local subdomain Ω_s as shown in Figure 3.1. This kind of method has advantages in higher adaptivity and lower cost in preparing input data for numerical analysis. It has attracted much attention due to their flexibility, and most importantly due to their ability in omitting the need for the human-labour intensive process of constructing geometric meshes [7]. Nodes can be simply added or deleted in desired position. The methods use also a local approximation to represent the trial function using the undetermined values (or the fictitious values) of the field variable at some randomly located nodes [8]. The moving least-squares (MLS) approximation is generally considered to be one of the best schemes to interpolate discrete data with excellent accuracy. The continuity of MLS approximation for approximation of unknown fields is given by the minimum between the continuity of the basis functions and that of the weight function [9]. So continuity can be tuned to a desired degree. In conventional discretization methods there is a discontinuity of secondary fields (gradients of primary fields) on the interface of elements. For modelling of continuously nonhomogeneous solids the approach based on piecewise continuous elements can bring some inaccuracies. Therefore, modelling based on C^1 continuity, like in meshless methods, is expected to be more accurate than conventional discretization techniques. The MLPG method is a fundamental base for the derivation of many meshless formulations, since trial and test functions can be chosen from different functional spaces. Previous results showed excellent convergence and accuracy of meshless approaches applied to many engineering problems of classical elasticity [7] [10] [11]. Recently, the MLPG method with a Heaviside step function as the test functions functions [7] [11] [12] [13] has been applied to solve two-dimensional (2D) homogeneous piezoelectric [14] and magnetoelectroelastic problems [15].

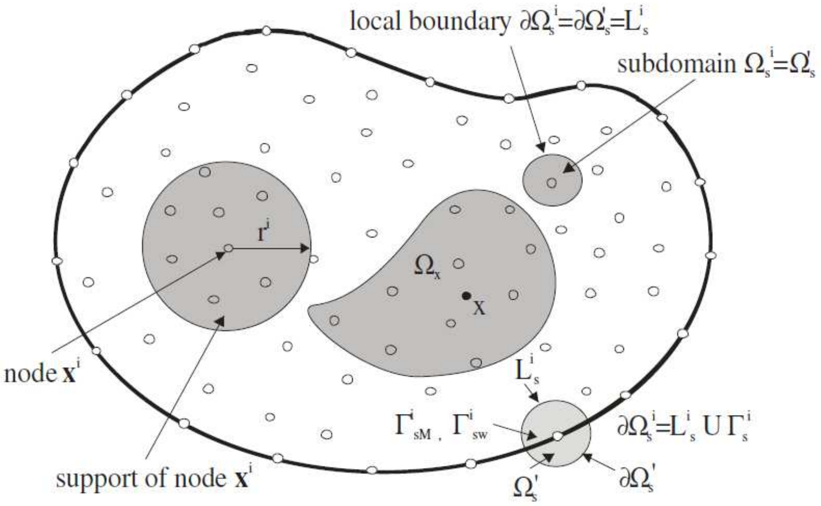


Figure 3.1 Local boundaries for weak formulation $\partial\Omega_s$, the domain Ω_x for MLS approximation of the trial function and the support area of weight function around node $\mathbf{x}^i$

The MLPG method based on the local weak form of the governing equations (2.4)-(2.6) of MEE material is written over local subdomain Ω_s. The local weak forms are then the starting point for deriving local integral equations with the use of Gauss divergence theorem and appropriate test function, in this case Heaviside unit step function. This finally yields following local integral equations

$$\int_{L_s+\Gamma_{su}} t_i(\mathbf{x},\tau)d\Gamma + \int_{\Gamma_{st}} T_i(\mathbf{x},\tau)d\Gamma + \int_{\Omega_s} X_i(\mathbf{x},\tau)d\Omega - \int_{\Omega_s} \rho(\mathbf{x})\ddot{u}_i(\mathbf{x},\tau)d\Omega = 0 \tag{3.1}$$

$$\int_{L_s+\Gamma_{sv}} q(\mathbf{x},\tau)d\Gamma - \int_{\Gamma_{sq}} Q(\mathbf{x},\tau)d\Gamma - \int_{\Omega_s} \Pi(\mathbf{x},\tau)d\Omega = 0 \tag{3.2}$$

$$\int_{L_s+\Gamma_{sa}} s(\mathbf{x},\tau)d\Gamma - \int_{\Gamma_{sb}} S(\mathbf{x},\tau)d\Gamma = 0. \tag{3.3}$$

where Ω_s is the local subdomain. L_s denotes the local boundary that is totally inside the global domain. Γ_{st}, Γ_{sq} and Γ_{sb} are the parts of the local boundary which coincide with the global traction vector T_i, normal component of the electric displacement vector Q and normal component of the magnetic induction vector S boundaries, respectively. Similarly, Γ_{su}, Γ_{sv} and Γ_{sa} are the parts of the local boundary that coincides with the global displacement, electric potential and magnetic potential boundaries, respectively.

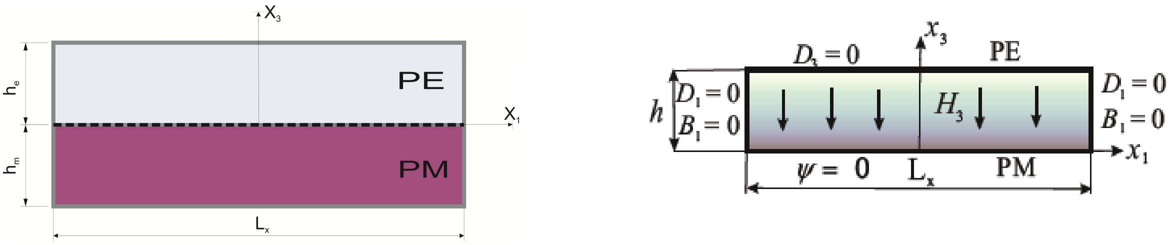
According to the MLS method, the approximation of unknowns can be given as

$$u^h(\mathbf{x},\tau)=\sum_{a=1}^{n}\phi^a(\mathbf{x})\hat{u}^a(\tau),\ u_{,j}^h(\mathbf{x},\tau)=\sum_{a=1}^{n}\phi_{,j}^a(\mathbf{x})\hat{u}^a(\tau) \tag{3.4}$$

where the nodal values $\hat{u}^a(\tau)$ are fictitious parameters for unknowns at time τ and $\phi^a(\mathbf{x})$ is the shape function associated with the node *a*. Collecting the discretized local integral equations together with the discretized boundary conditions, one obtains a complete system of ordinary differential equations (ODE) which can be solved by the Houbolt method.

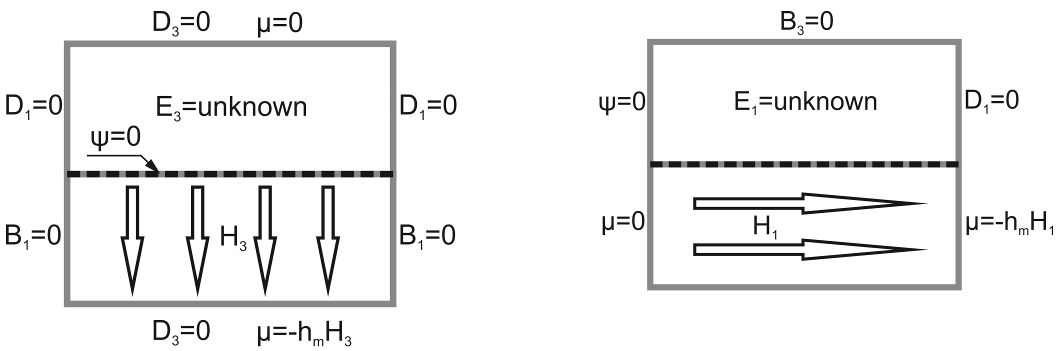
4. Enhancement of the magneto-electroelastic coefficient

A strong ME effect has been recently observed by Pan and Wang [16] in artificially fabricated multiferroic composites. Thus, the 2D layered (Figure 4.1a) and FGM (Figure 4.1b) composites with a pure piezomagnetic behaviour on its bottom surface and a pure piezoelectric one on its top surface are investigated.



a) b)

Figure 4.1 Geometry of: a) two-layered multiferroic composite; b) FGM multiferroic



a) b)

Figure 4.2 The longitudinal ME effect: a) out-of-plane; b) in-plane

In case of layered composite, it is needed to simulate jumps in the secondary field. Two sets of collocation nodes are assigned on both the +side and the –side of the material interface at the same location, but with different material properties. The MLS approximations are carried out separately on particular sets of nodes within each of the homogeneous domains.

Therefore, the high order continuity is kept within each homogeneous part, but not across their interface.

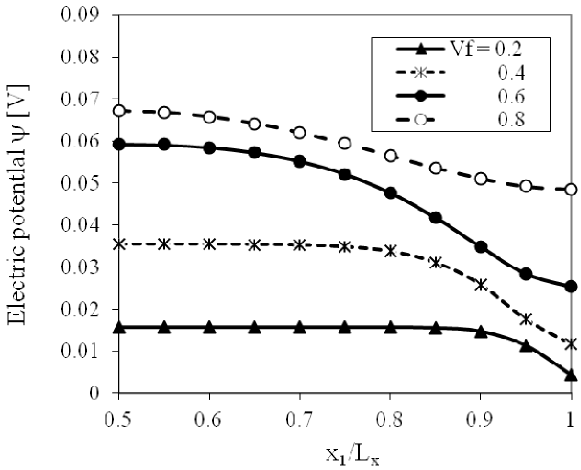


Figure 4.3 Variation of the electric potential along x_1 on the top surface of PE layer

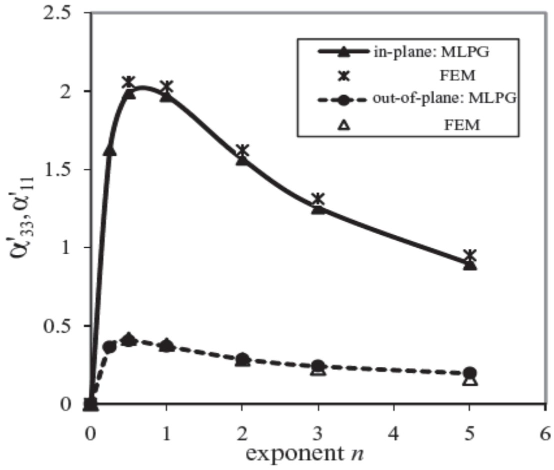


Figure 4.4 Variation of the magnetoelectric coefficient versus the volumetric ratio

In the numerical examples, bottom of the piezomagnetic layer is fixed $u_3 = 0$. Magnetic potential $\mu = -0.1A$ is also applied on the bottom surface of the PM layer. In the layered composite, the volumetric ratio of the PE layer is introduced as $V_f = h_e/(h_e + h_m)$, where the piezoelectric layer (h_e) is changed and the piezomagnetic layer (h_m) is unchanged. One can observe the influence of the volumetric ratio of the PE/PM layers on the electric potential on the top surface of the PE layer in Figure 4.3. With increasing PE layer thickness the electric potential on the top surface increases. In case of the FGM multiferroic, material properties are continuously varying along the plate thickness using polynomial distribution. Both out-of-plane $(\bar{\alpha}_{33})$ and in-plane $(\bar{\alpha}_{11})$ magnetoelectric coefficients are presented in Figure 4.4. One can observe that the maximum ME coefficient values are obtained for exponent value of about 0.5. The in-plane ME coefficient $(\bar{\alpha}_{11})$ is significantly larger than the out-of-plane ME coefficient $(\bar{\alpha}_{33})$. Recently, Pan and Wang [16] observed a similar phenomenon for layered multiferroic composites, where the in-plane ME coefficient was about 2 times larger than the out-of-plane one. In considered FGM multiferroic composites, one can see more enhanced in-plane ME coefficient.

5. Circular MEE plates with FGM properties

Many piezoelectric and magnetoelectroelastic components have plate-like shapes, often circular. Some of these components like resonators and filters are used for the purpose of frequency control. Therefore, the MLPG method is applied for circular plate bending analyses assuming also functionally graded magnetoelectroelastic material properties. Using the cylindrical coordinates $\mathbf{x} = [r, \theta, z]$, 3D axisymmetric body is reduced to a 2D problem with unknowns on the cross-sectional domain as shown in Figure 5.1. Therefore, nodal points are introduced and spread on the analyzed 2D domain.

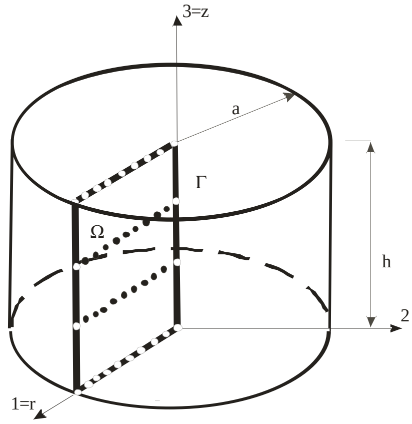


Figure 5.1 Geometry and generation of a circular plate

In the cylindrical coordinate system, the governing equations (2.4)-(2.6) take the form

$$\sigma_{rr,r}(r,z,\tau) + \sigma_{rz,z}(r,z,\tau) + \frac{1}{r}\left[\sigma_{rr}(r,z,\tau) - \sigma_{\varphi\varphi}(r,z,\tau)\right] - \rho(\mathbf{x})\ddot{u}_r(r,z,\tau) = -X_r(r,z,\tau) \quad (5.1a)$$

$$\sigma_{rz,r}(r,z,\tau) + \sigma_{zz,z}(r,z,\tau) + \frac{1}{r}\sigma_{rz}(r,z,\tau) - \rho(\mathbf{x})\ddot{u}_z(r,z,\tau) = -X_z(r,z,\tau) \quad (5.1b)$$

$$D_{r,r}(r,z,\tau) + D_{z,z}(r,z,\tau) + \frac{1}{r}D_r(r,z,\tau) = 0 \quad (5.2)$$

$$B_{r,r}(r,z,\tau) + B_{z,z}(r,z,\tau) + \frac{1}{r}B_r(r,z,\tau) = 0 \;. \quad (5.3)$$

Then, local boundary-domain integral equations (3.1)-(3.3) are converted into

$$\int_{\partial\Omega_s} \sigma_{rb}(r,z,\tau)n_b d\Gamma + \int_{\Omega_s} \frac{1}{r}\left(\sigma_{rr} - \sigma_{\varphi\varphi}\right)d\Omega - \int_{\Omega_s} \rho(\mathbf{x})\ddot{u}_r(r,z,\tau)d\Omega = -\int_{\Omega_s} X_r(r,z,\tau)d\Omega \quad (5.4a)$$

$$\int_{\partial\Omega_s} \sigma_{zb}(r,z,\tau)n_b d\Gamma + \int_{\Omega_s} \frac{1}{r}\sigma_{rz}(r,z,\tau)d\Omega - \int_{\Omega_s} \rho(\mathbf{x})\ddot{u}_z(r,z,\tau)d\Omega = -\int_{\Omega_s} X_z(r,z,\tau)d\Omega \quad (5.4b)$$

$$\int_{\partial\Omega_s} D_b(r,z,\tau)n_b d\Gamma + \int_{\Omega_s} \frac{1}{r}D_r(r,z,\tau)d\Omega = 0 \quad (5.5)$$

$$\int_{\partial\Omega_s} B_b(r,z,\tau) n_b d\Gamma + \int_{\Omega_s} \frac{1}{r} B_r(r,z,\tau) d\Omega = 0 \tag{5.6}$$

In the numerical example, the plate is loaded by stationary mechanical tension on the top plate surface. An exponential variation of material coefficients is assumed, where two exponential coefficients $\gamma = 6.9315$ and 4.054 are considered. For the first exponential coefficient the material parameters on the top surface of the plate are doubled with respect to those on the bottom plate surface. The numerical results of variations of the deflections for the clamped plate with radial coordinate are given in Figure 5.2. The numerical results are compared with their homogeneous counterpart. Since the mechanical parameters for the FGM plate are larger than those in a homogeneous case, the mechanical deflection is reduced for the FGM plate. A larger reduction is observed for a larger gradation parameter. The variation of the induced electrical potential on the top surface of the plate with the radial coordinate is presented in Figure 5.3. The value of the electric potential is almost uniform over the large part of the radius from the plate centre for the homogeneous case. However, for the FGM plate the electrical potential is gradually growing with reaching the maximum value at the plate centre. One can observe that the electrical potential in the FGM plate is significantly larger than in a homogeneous plate. Therefore, FGM magnetoelectroelastic plates are more convenient for sensing of deformations than homogeneous ones.

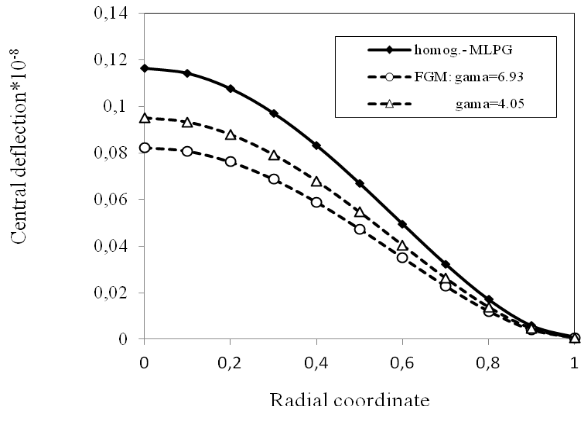


Figure 5.2 Variation of deflections with the radial coordinate for clamped magnetoelectroelastic plates

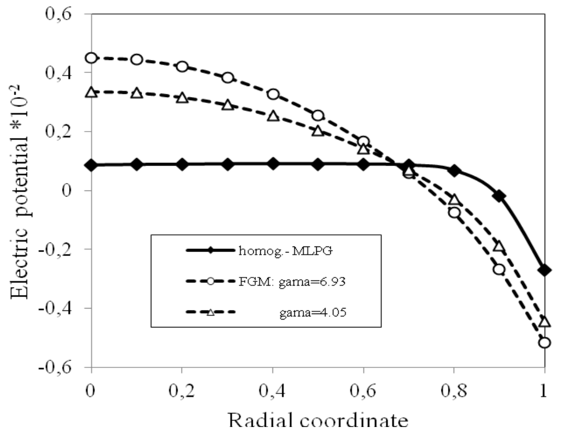


Figure 5.3 Variation of el. potentials on the top surface with the radial coordinate for clamped magnetoelectroelastic plates

6. Large deflections of rectangular MEE plates

A plate of total thickness *h* with homogeneous MEE material properties and with its mean surface occupying the domain Ω in the plane (x_1, x_2) is considered. The axis $x_3 = z$ is perpendicular to the mid-plane (Figure 6.1) and the origin is located at the bottom of the plate.

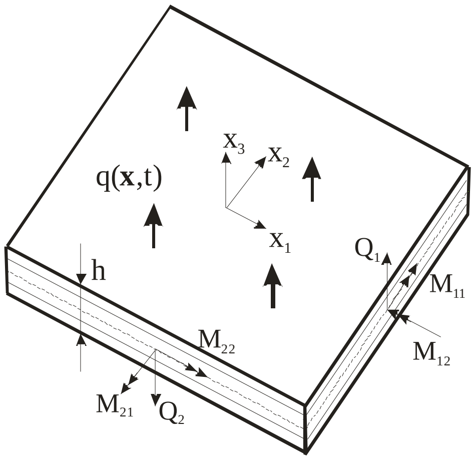


Figure 6.1 Sign convention of bending moments and forces for a plate

The MLPG method is proposed to solve problems for thick MEE plates under a large deformation described by the Reissner– Mindlin theory. The electric and magnetic field components are assumed to be zero in the in-plane directions of the plate. A quadratic power-expansion of the electric and magnetic potentials in the thickness direction of the plate is considered. The bending moment, normal and shear force expressions are obtained by integration through the plate for the considered constitutive equations. The Reissner–Mindlin governing equations of motion are subsequently solved for a time-harmonic plate bending problem. The Reissner– Mindlin theory reduces the original 3D thick plate problem to a 2D problem. In the used meshless method, nodal points are randomly distributed over the neutral plane of the considered plate.

Time-harmonic load is a special case of the general dynamic analysis. Time variation of physical fields is given by the frequency of oscillation ω. Then the governing equations for the amplitudes have the following form [17][18]:

$$M_{\alpha\beta,\beta}(\mathbf{x},\omega) - Q^I_\alpha(\mathbf{x},\omega) = -I_M \omega^2 w_\alpha(\mathbf{x},\omega) \tag{6.1}$$

$$Q^I_{\alpha,\alpha}(\mathbf{x},\omega) + \left(T_{\alpha\beta}(\mathbf{x},\omega) w_{3,\beta}(\mathbf{x},\omega)\right)_{,\alpha} + q(\mathbf{x},\omega) = -I_Q \omega^2 w_3(\mathbf{x},\omega) \tag{6.2}$$

$$T_{\alpha\beta,\beta}(\mathbf{x},\omega) + q_\alpha(\mathbf{x},\omega) = -I_Q \omega^2 u_{\alpha 0}(\mathbf{x},\omega)\,, \qquad \mathbf{x} \in \Omega \tag{6.3}$$

and the additional set of two governing equations is given by Maxwell equations as

$$e_{31}w_{1,1}(\mathbf{x},\omega)+e_{32}w_{2,2}(\mathbf{x},\omega)-2h_{33}\frac{\psi_2(\mathbf{x},\omega)}{h^2}-2\alpha_{33}\frac{\mu_2(\mathbf{x},\omega)}{h^2}=0 \tag{6.4}$$

$$d_{31}w_{1,1}(\mathbf{x},\omega)+d_{32}w_{2,2}(\mathbf{x},\omega)-2\alpha_{33}\frac{\psi_2(\mathbf{x},\tau)}{h^2}-2\gamma_{33}\frac{\mu_2(\mathbf{x},\tau)}{h^2}=0 . \tag{6.5}$$

where $M_{\alpha\beta}$, Q_α^I , $T_{\alpha\beta}$ are the bending moments, the linear part of the shear force and normal forces, respectively, with indices α,β=1,2. w_α and w_3 are rotation around x_α-axis and out-of-plane deflection, respectively. A transversal load is denoted by $q(\mathbf{x},\tau)$, and $q_\alpha(\mathbf{x},\tau)$ represents the in-plane load. If the mass density is constant throughout the plate thickness, one can obtain

$I_M=\frac{\rho h^3}{12}$, $I_Q=\rho h$.

Applying the Gauss divergence theorem to the governing equations (6.1)-(6.3) in the local weak-form and choosing the unit step functions for the test functions in each subdomain, then, the local weak-forms are transformed into the following local integral equations (LIEs)

$$\int_{\partial\Omega_s} M_\alpha(\mathbf{x},\omega)d\Gamma-\int_{\Omega_s} Q_\alpha^I(\mathbf{x},\omega)d\Omega+\int_{\Omega_s} I_M\omega^2 w_\alpha(\mathbf{x},\omega)d\Omega=0 \tag{6.6}$$

$$\int_{\partial\Omega_s} Q_\alpha^I(\mathbf{x},\omega)n_\alpha(\mathbf{x})d\Gamma+\int_{\Omega_s} I_Q\omega^2 w_3(\mathbf{x},\omega)d\Omega+\int_{\partial\Omega_s} T_\alpha(\mathbf{x},\omega)w_{3,\beta}(\mathbf{x},\omega)d\Gamma+\int_{\Omega_s} q(\mathbf{x},\omega)d\Omega=0 \tag{6.7}$$

$$\int_{\partial\Omega_s} T_\alpha(\mathbf{x},\omega)d\Gamma+\int_{\Omega_s} q_\alpha(\mathbf{x},\omega)d\Omega+\int_{\Omega_s} I_Q\omega^2 u_{\alpha 0}(\mathbf{x},\omega)d\Omega=0 \tag{6.8}$$

where $\partial\Omega_s$ is the boundary of the local subdomain, $M_\alpha(\mathbf{x},\omega)=M_{\alpha\beta}(\mathbf{x},\omega)n_\beta(\mathbf{x})$ and $T_\alpha(\mathbf{x},\omega)=T_{\alpha\beta}(\mathbf{x},\omega)n_\beta(\mathbf{x})$ are the normal bending moment and the traction vector, respectively. n_α stands for the unit outward normal vector to the boundary $\partial\Omega_s$.

In order to linearize the problem, the nonlinear terms will be considered in the local integral equations (LIE) iteratively. It means that nonlinear terms computed in the *(k-1)*th iteration are considered in the LIE for *k*th iteration. Insertion of the MLS-discretized moment, tractions and shear force fields into the local integral equations (6.6) - (6.8) yields the discretized local integral equations. If the MEE plate is used as a sensor, the plate is under a mechanical load. Then, the system of the discretized LIE has to be supplemented by Eqs. (6.4)-(6.5). In the *k*th iteration step the linearized boundary value problem is resolved. The iteration process is stopped if the differences between the deflections in two consecutive steps are less than the prescribed tolerance.

In the numerical examples, a square MEE plate with a side-length $a = 0.254m$ is analyzed to verify the proposed computational method. The total thickness of the plate is $h = 0.012m$. On the top surface a uniform mechanical load is applied. Simply supported boundary conditions are considered.

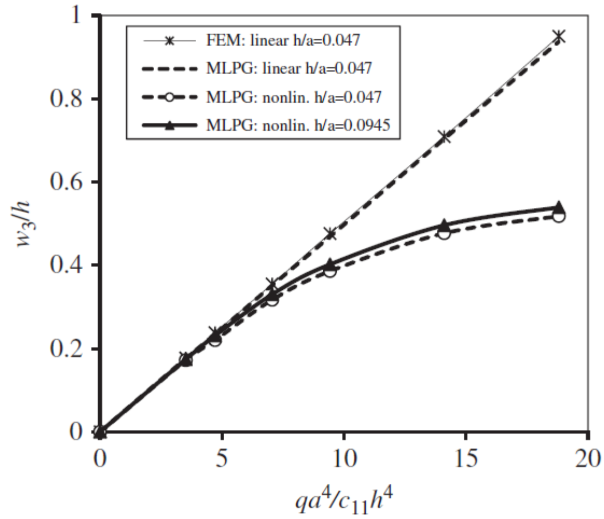


Figure 6.2 Variation of the plate deflection at the centre of a simply supported plate

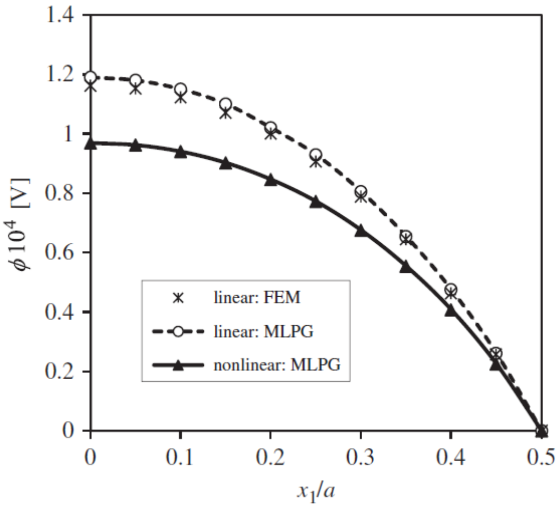


Figure 6.3 Variation of the electric potential along the x_1-coordinate

The variation of the central plate deflection ($x_1 = x_2 = 0$) with the intensity load is presented in Figure 6.2. The intensity load is given by a nondimensional parameter $\tilde{q} = qa^4 / c_{11}h^4$. Two different plate thicknesses are considered here. The plate deflection w_3 is normalized by the plate thickness. It can be seen that the plate thickness has only a slight influence on the normalized deflections. Variations of the electric potential along the x_1-coordinate are presented in Figure 6.3 at the nondimensional intensity load $\tilde{q} = 9.42$. At this load intensity one can see clearly that nonlinear effect is apparent. Furthermore the difference of the induced electrical potentials based on the linear and nonlinear theory is more than 20%. The maximum electric potential for the simply supported plate is reached at the centre of the plate. The magnetic potential at the plate centre is proportional to the electric potential with $\phi / \mu = 0.61 \cdot 10^{-2}$ and this value corresponds to the material parameters ratio $d_{31}h_{33} / e_{31}\gamma_{33}$.

7. Cracks in MEE solids

The MEE materials can be used as sensors and actuators for active vibration control of various elastic structures. In this kind of the structures, an interface is formed between elastic and magnetoelectroelastic material. The material discontinuity in laminated composite materials leads to large interlaminar stresses and the possibility of initiation and propagation of cracks. For this reason, a crack analysis of an interface between two dissimilar materials is important.

The meshless local Petrov-Galerkin (MLPG) method is applied to solve the boundary value problem by same way as in case of two layered composite in Chapter 4. The intensity factors can be computed as an interface crack between two dissimilar anisotropic and linear magnetoelectroelastic materials by the generalized crack-opening displacements (CODs) [19] as

$$\Delta\mathbf{u}(r) = (\mathbf{H}+\bar{\mathbf{H}})\sqrt{\frac{r}{2\pi}}\left[\frac{Kr^{i\varepsilon_1}\mathbf{w}}{(1+2i\varepsilon_1)\cosh(\pi\varepsilon_1)} + \frac{\bar{K}r^{-i\varepsilon_1}\bar{\mathbf{w}}}{(1-2i\varepsilon_1)\cosh(\pi\varepsilon_1)} + \frac{K_{IV}r^{-\varepsilon_2}\mathbf{w}_4}{(1-2\varepsilon_2)\cosh(\pi\varepsilon_2)} + K_V\mathbf{w}_5\right] \tag{7.1}$$

where $K = K_I + iK_{II}$ is the complex stress intensity factor, K_{IV} is the electric displacement intensity factor and K_V is the magnetic induction intensity factor, an overbar denotes the complex conjugate and r is the distance from the crack-tip to measured data point. The complex Hermitian matrix $\mathbf{H}$ is determined by the material properties from the both layers and computed from the eigenvalue problem as well the bimaterial constants ε_1 and ε_2 and the eigenvectors $\mathbf{w}$, $\mathbf{w}_4$ and $\mathbf{w}_5$ are determined by the eigenvalue problem as shown in [19].

In numerical examples, a finite strip with the central interface crack is considered. The crack with length $2a = 1m$ along the axis x_1, the width of the strip $2w = 2a/0.4$ and height of each layer $h_I = h_{II} = 1.2w$ is assumed. The strip is under static mechanical load $\sigma_0 = 10^8 Pa$ on the top of MEE layer. The results for the crack-displacements, which are computed on both crack-faces are presented in Figure 7.1. Larger crack-opening-displacement on the lower crack-face due to smaller material stiffness parameters is clearly indicated. The variations of the electrical and magnetic potentials along the crack-faces are given in Figure 7.3 only for upper crack face, which corresponds to MEE material. Intensity factors are computed from general crack opening displacements. The value of stress intensity factor $K_I = 1.91 \cdot 10^8 Pam^{1/2}$ is significantly larger in comparison with $K_I = 1.4 \cdot 10^8 Pam^{1/2}$ [20] that corresponds to the strip of same geometry but homogenous MEE material properties in entire solid.

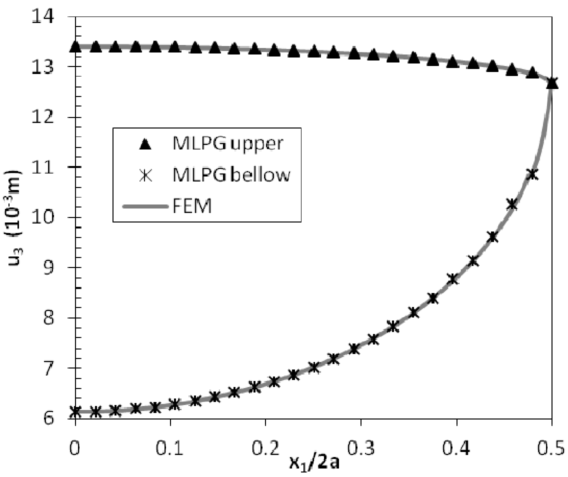


Figure 7.1 The crack displacement under mechanical loading

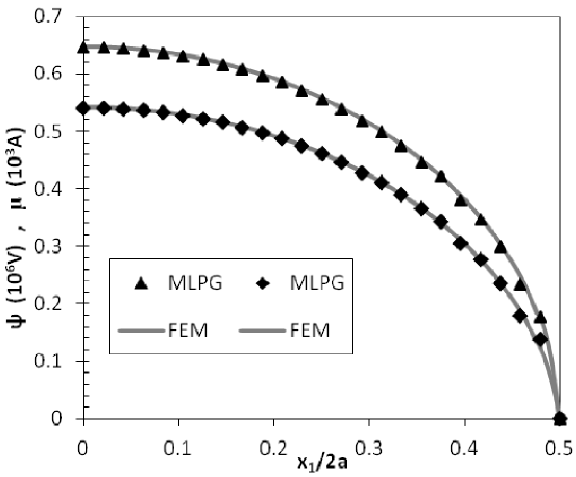


Figure 7.2 The electrical and magnetic potentials on the crack face of MEE layer under mechanical load

8. CONCLUDING REMARKS

The meshless local Petrov-Galerkin method (MLPG) method is a numerical method which offers easy preparation of input data, because no element mesh data are required and provides with more flexibility, because it allows straightforward implementation of problems described by PDE with variable coefficients, compared to classical computational methods. The proposed method is a truly meshless method, which requires neither domain elements nor background cells in either the interpolation or the integration. In all mentioned analyses, nodal points are distributed over the investigated domains. No finite elements are required. Each node is the center of a circle surrounding this node. Therefore the analyzed domain is divided into small overlapping circular subdomains. The weak-form on small subdomains with a Heaviside step function as the test functions is applied to derive local integral equations. After performing the spatial MLS approximation, a system of ordinary differential equations for certain nodal unknowns is obtained in case of transient analysis. Then, the system of the ordinary differential equations of the second order resulting from the equations of motion is solved by the Houbolt finite-difference scheme as a time-stepping method. The present computational method based on local integral equations with a meshless approximation seems to be very promising for presented coupled field analyses. Majority of numerical results are compared with the results obtained by the FEM with good agreement.

The applications of magnetoelectroelastic materials to smart structures have certain advantages in comparison to piezoelectric materials. Less material could be used for producing the strains of same magnitude. In the Chapter 4, two-layered composite consisting of two dissimilar piezoelectric (PE) and piezomagnetic(PM) materials with thickness h_e and h_m, respectively, is investigated by MLPG method. The ME voltage coefficient increases with increasing thickness ratio h_m/h_e, between the magnetostrictive and the piezoelectric layers because the compressive stress is higher in thinner piezoelectric layer. Magnetoelectric coefficients for functionally graded material parameters in entire composite plate are also investigated. The parameters are continuously varying along the plate thickness using polynomial distribution. The magnetic intensity or mechanical load causes deformations in the piezomagnetic constituent of the FGM composite. The composite plate deformations induce electric potential at the piezoelectric constituent of the FGM composite. At an optimal gradation of the piezoelectric or piezomagnetic constituents one can obtain a significant enhancement of the magnetoelectric coefficient. The in-plane coefficient is significantly larger than the out-plane one. Understanding of the FGM plate gives a chance to design smart structures with an optimal magnetoelectric coefficient.

In the Chapter 5, the MLPG is applied to a circular plate with functionally graded magneto-electro-elastic material properties. Due to axial symmetry it is sufficient to analyze only the cross section of the plate. Numerical results for clamped plates showed that induced electrical potential is significantly enhanced in FGM plates. Therefore, FGM plates are convenient for sensing mechanical deformations. Further research to optimize FG material properties is needed, to get a strong electrical signal from smart MEE structures.

The MLPG method is proposed for nonlinear large-deflections of MEE plates under mechanical and electrical loads in Chapter 6. Von Karman’s theory of large deflections is applied for Reissner–Mindlin plates with MEE properties. If a quadratic variation of the electric and magnetic potentials along the plate thickness is assumed, the original 3D thick plate problem is reduced to a 2D problem. Nodal points are randomly distributed over the mean plane of the considered plate. Numerical results showed that coupling material parameters have a vanishing influence on the plate deflection under a pure mechanical load. Also their influence on the eigen-frequencies is vanishing. Induced electric and magnetic potentials are lower based on the nonlinear large-deformation theory than those in the corresponding linear case; however, their spatial variations in both cases are similar.

Interesting subject of a study are crack problems. Chapter 7 analyzes composite strip that consists of magnetoelectroelastic layer on the top and elastic layer on the bottom. A central crack is considered on the interface. MEE layer on the top is intended to act as an actuator and elastic layer on the bottom is clamped. On the crack face of elastic layer, large crack opening displacement is observed only under mechanical load. The stress intensity factor is significant larger than one in MEE materials considering the entire domain.

Numerical results presented in this thesis have shown unique properties of the meshless MLPG method for the analysis of engineering structures with MEE material properties. Focusing on nodes instead of finite elements bring certain advantages, especially in the analysis of FGMs. Developed numerical solutions and computational procedures may contribute to broader application of MEE materials and also facilitate the analysis of complex engineering problems.

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