1 INTRODUCTION

Among engineering vibration tasks, testing bridges under the effect of a moving load is an old problem. Before the appearance of computers, testing of bridges under the effect of a moving load could be only analysed through essential simplifications. Timoshenko (1955) gave one possible simplification but ignored the mass of a moving body on a beam. One possible simplification is to ignore the structure’s mass and another is to ignore the mass of the moving body. Frýba (1972) analysed the simultaneous effect of load and mass. In these tests, the bridge was simulated by a beam of constant stiffness and tested as a continuum.

In the application of the finite element method for computing the matrices of differential equations of a constant coefficient for the given starting conditions, several methods have been developed including the Wilson-Θ method described by Bathe & Wilson (1976). If we also want to take into consideration the mass of the moving body during tests, a quasi-harmonic vibration task should be solved. Upon the calculation of the moving mass the matrix of differential equations has time-dependent coefficients. We can see this in the work of Inbanathan & Wieland (1987). Later, as an advance, the vehicle was simulated not only as a moving mass but also as a dynamic system with several degrees of freedom - as Green and Cebon (1994) did for road vehicles.

However, it is well known that the dynamic overload is greatly affected by damping, too. The known methods - reported, e.g., by Olsson (1985) - are able to compute the effect of the external damping proportional to the velocity; however, they are not appropriate for taking the structural damping into consideration. Györgyi (1996) suggested a method for computing the structural damping as an equivalent external damping for which, as preparation, the eigenvalue problem belonging to the undamped dynamic system of several degrees of freedom must be solved.

J. Györgyi, Assoc. Prof., József Györgyi, born 1941, Civil engineer-1964, Specialist engineer of engineering-mathematics-1972, Candidate of Technical Science-1985, Dr. Habil. 2001, associate professor, Budapest University of Technology and Economics, Department of Structural Mechanics. Main research topics: Dynamics of Structures (Earthquake analysis, Moving forces on structures, Impact problems, Wind effect)
Address: H-1221 Budapest Arany János u. 96/b, Phone/Fax: 36 1 2262096, e-mail: gyorgyi@ep-mech.mec.bme.hu

G. Szabó, civil engineer, PhD student Gergely Szabó, born 1980, received his civil engineering degree from University of Technology Budapest-2005, PhD Student, Main research topics: Dynamic calculation of bridges (Moving forces on structures, Wind effect).

ABSTRACT

If we want to consider the dynamic effect of a moving vehicle during the test of a structure, we have the possibility of constructing a model. Our first model involved calculations with force groups; during the analysis we wanted to calculate the frequency-independent structural damping and use a modal analysis. If we know the distance between forces, we can calculate the critical force group’s velocity. Our second model was calculated by moving the mass points. In this case the mass matrix in the dynamic matrix equation of the bridge was time-dependent. Using the eigenvectors of an undamped system, we can apply a quasi-modal analysis. With this model, the group force’s critical velocity was less, and the dynamic effect was higher. Our third model was when we calculated the interaction of a bridge and vehicle system. The results show the important role of the dynamic characteristics of a bridge and the velocity of a train.

KEY WORDS

• train-bridge interaction,
• critical force group’s velocity,
• quasi-modal analysis

If we generate some of the smallest eigenvectors both for the structure and the vehicle, we can use the technique of quasi-modal analysis. This method, which was published by Györgyi (1995-1996), allows for the simultaneous analysis of both the mass and stiffness matrix changing over time and the external damping proportional to the velocity and the frequency-independent structural damping modal analysis. Györgyi (1999, 2005) presented the numerical results of the calculations in the case of a railway moving on a beam bridge. In their calculations for an arc bridge, Györgyi and Szabó (2006a) analysed a different kind of train. The effect of the application of a two-dimensional beam and three-dimensional structural model was shown by Györgyi and Szabó (2006b). In this paper we can see the numerical results of the calculations with different parameters of the vehicle’s stiffness and the ballast.

2 CALCULATION OF THE DYNAMIC EFFECT ON A SIMPLY SUPPORTED BEAM

The dynamic response of a beam to a moving force [1] with structural damping [2] will be:

$$v(x,t) = \sum_{\gamma} \phi_{\gamma} \frac{1}{\omega_{\gamma}} \left[ \frac{\gamma^2}{\omega_{\gamma}^2} + \frac{\gamma^2}{\omega_{\gamma}^2} \right] + \sin(\omega_{\gamma} t - \varphi)$$

The damping parameter can be calculated from the percent damping

$$\gamma = \frac{\omega_{\gamma}}{50}, \quad \sigma = \frac{r \pi v}{2}, \quad \alpha_{\gamma} = \frac{r^2 \pi^2}{2} \sqrt{\frac{\mu}{E}},$$

where $r$ is the $r$-th natural circular frequency of the beam. The $\varphi = \arctan \frac{\sqrt{k m \sigma}}{k - m \sigma}$ and $\alpha_{\gamma} = \sqrt{1 - \frac{\gamma^2}{4}}$. The vibration of the bridge continues if the force is not already on the bridge. The free vibration in this case is:

$$v(x,t) = \sum_{\gamma} \frac{1}{2 \mu} \left[ \frac{\gamma^2}{\omega_{\gamma}^2} \right] + \sin(\omega_{\gamma} t - \varphi)$$

![Fig. 1 Dynamic influence line at the middle point of a beam](image1)

![Fig. 2 Force group with the same distance over an influence line](image2)

![Fig. 3. Displacements of the mid-point of a bridge from moving force groups at different velocities](image3)
3 ANALYSIS OF AN ARC BRIDGE

It is very important to stress that the analytical methods can be applied in simpler cases only. If the structure is more complex than a simply supported beam, i.e., we want to model a train with moving forces and mass points, numerical methods need to be used.

3.1 Modelling a bridge

The investigated bridge structure is a steel arch railway bridge with a span of 52.0m and a height of 9.0m. The main dimensions of the bridge can be seen in Fig. 4.

The main elements are the two arches and the two main girders. The span is 52.00 m. The main girders are supported by cables with a diameter of 60 mm. The distance between these cables is 4m. The ballast bed passes through the bridge; therefore, a steel plate is necessary to support it. There are also secondary structural elements such as cross girders with a distance of two meters from each other and six longitudinal trapezoidal bars. The bridge carries a single track. During the dynamic analysis we investigated the vertical displacements of the quarter point and the middle point of the main girder. We constructed three different models of the bridge. The first one is the simplest, i.e., a two-dimensional beam model (Fig. 5.). The main girder and the arch are modelled with two-dimensional beams with three degrees of freedom per node. The cables are modelled with two-dimensional link elements with tensional capacities only. The half weight of the deck elements and the ballast bed were reduced on the main girder.

The second and third models are more complex. The second one is a three-dimensional beam model, whereas the first model was developed through modelling the cross girders and the longitudinal bars. In this case we applied symmetrical conditions. The weight of the ballast bed was reduced on the cross girders. These models can be seen in Fig. 6. The third one is the most difficult model. This model was constructed by using shell elements with six degrees of freedom per node. The cables are two-dimensional bars.

3.2 Modelling the vehicle

We used two types of trains. At first we used a Russian diesel locomotive (M62) as a real train (Fig. 7) on this railway line. This engine is considered a low-speed train with a top speed of 105 km/h.
We put a number of cement carrier trucks together with a diesel engine to model a realistic train. Secondly, we used a Japanese express train as a dynamic load, which is fictive in Hungary, but can be necessary to model really high-speed moving loads. The train consists of 12 railcars. This vehicle and its brief parameters are in Fig. 8. The length of a railcar is 25m.

We can model these trains in three ways, i.e., as moving forces, moving forces and mass points together, and a moving vehicle system (Fig. 9.).

4 DYNAMIC CHARACTERISTICS OF THE BRIDGE

In order to understand the dynamic behaviour of an arch bridge due to a moving train, we had to determine the mode shapes, which can be seen in Fig. 10. If we compare these shapes with the mode shapes of a simply supported beam, we find that while the first shape of a simple beam is symmetrical, in the case of an arch bridge, it is
asymmetrical (Fig. 10). Moreover, the frequencies are much closer to each other in the case of an arch (Table 1). Therefore, the dynamic behaviour of a simply supported beam bridge and an arch bridge due to a vehicle load is quite different.

We compared the required number of eigenvectors for the proper degree of accuracy by doing a time step dynamic analysis which will be introduced in 4 (Tab. 1). We found that for the same degree of accuracy, we needed many more eigenvectors. By monitoring the displacements, we compared the convergence. If we consider the bending moments or the shear forces of the girders, we need even more eigenvectors.

The difference between the behaviour of a simply supported beam bridge and an arc bridge can be determined by doing a simple static analysis. The deformation due to total loading is symmetrical in both cases, but due to the asymmetrical loading, the deformation of the beam is almost symmetrical, while in the case of the arc bridge, it is mostly asymmetrical. This type of behaviour will be determined in the results.

5 Mathematical Methods Applied

The main purpose of our research was to compute the response of the bridge due to dynamic loading. It is necessary to solve the following matrix equation of motion:

$$\begin{bmatrix} M + M(t) \end{bmatrix} \ddot{\mathbf{x}}(t) + C \dot{\mathbf{x}}(t) + K \mathbf{x}(t) = \mathbf{q}(t)$$

(3)

If we want to take into account the weight of the moving train, a time-dependent mass matrix component appears. Since the mass of the train is moving on the bridge, the dynamic characteristics of the bridge - vehicle system is changing over time, so there are no natural frequencies. We applied a quasi-modal analysis, - see Györgyi (1995-1996) - to look for the solution in the form: $\mathbf{x} = \mathbf{V} \mathbf{y}$.

We have to solve the following equation of motion at each time step using the Wilson-θ method:

$$\begin{bmatrix} \mathbf{D} + \frac{6}{\Theta \Delta t} \mathbf{b},_{\mathbf{x}} \mathbf{y} + \frac{6}{\Theta \Delta t} \mathbf{y}_{r_{\mathbf{x}}+\mathbf{y}_r} + \frac{\mathbf{F}}{\Theta} + \frac{3}{\Theta \Delta t} \mathbf{y}_r + 2 \mathbf{y}_{r_{\mathbf{x}}+\mathbf{y}_r} \end{bmatrix} \mathbf{y}_{r_{\mathbf{x}}+\mathbf{y}_r} = \mathbf{y}_{r_{\mathbf{x}}} \cdot (\mathbf{F} - \mathbf{G})$$

(4)

For the next time step:

$$\begin{align*}
\mathbf{y}_{r_{\mathbf{x}}} &= \mathbf{y}_r + \frac{\Delta \mathbf{y}}{2} (\mathbf{y}_{r_{\mathbf{x}}} + \mathbf{y}_r), \\
\mathbf{y}_{r_{\mathbf{x}}} &= \mathbf{y}_r + \Delta \mathbf{y}_r + \frac{\Delta t}{6} (\mathbf{y}_{r_{\mathbf{x}}} + 2 \mathbf{y}_r),
\end{align*}$$

Where: $f(t) = \mathbf{V}' \mathbf{q}(t)$,

$$\begin{align*}
f_{r_{\mathbf{x}}} &= f_r + \Theta f_{r_{\mathbf{x}} + f_r},
\end{align*}$$

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
 & \text{simple beam} & & & \text{arc bridge} & \\
\hline
m & freq (Hz) & $U_{\text{max}}$ & m & freq (Hz) & $U_{\text{max}}$ \\
\hline
1 & 4.02 & 0.02124 & 1 & 2.65 & 0.02145 \\
2 & 15.92 & 0.02124 & 2 & 7.36 & 0.02334 \\
3 & 35.21 & 0.02122 & 3 & 6.21 & 0.02547 \\
4 & 61.18 & 0.02122 & 4 & 10.62 & 0.02541 \\
5 & 93.06 & 0.02122 & 5 & 11.03 & 0.02558 \\
6 & 130.09 & 0.02122 & 6 & 16.61 & 0.02557 \\
7 & 171.67 & 0.02122 & 7 & 23.70 & 0.02558 \\
\hline
\end{tabular}
\caption{Natural frequencies and convergences}
\end{table}
The $\Theta$ is a stability parameter ($\Theta = 1.4$); $\Delta t$ is the time step; $q(t)$ is the time dependent load vector; $M(t)$ is the time dependent mass matrix; $V$ contains the eigenvectors of the undamped system calculated using Ansys software. The $y_{t + \Delta t}$ can be obtained (4), and from that we can calculate the $y_{t + \Delta t}$ vector and the displacements ($x_{t + \Delta t} = V y_{t + \Delta t}$). If the mass matrix is time dependent, we can get the mode shapes calculated with Ansys and use them in the (3) algorithm written using Matlab software. If the mass matrix is constant when calculating only with the forces, the calculation can be carried out using the Ansys software only.

6 RESULTS

During the dynamic analysis we focused only on the vertical displacements of two points of the main girder, i.e., the quarter and middle points.

6.1 Low speed train

6.1.1 A single engine

At first the loading is the M62 engine alone. In Figs. 11 and 12, the vertical displacements of the main girder can be seen. In order that the role of the ballast bed can be determined, we calculated without (Fig. 11) and with the (Fig. 12) ballast bed. We constructed the 2D finite element model in two ways. At first, the weight of the ballast was not reduced on the main girder, but it was reduced the second time. The circular frequencies are consequently different from each other (without the ballast bed: $\omega_1 = 4.89$Hz and with the ballast bed: $\omega_1 = 2.65$Hz). We also investigated the weight of the train using the Wilson method. We applied the method at the top speed of the train ($v_{max} = 29.1$ m/s).

The main difference between Figs. 11 and 12 is that the presence of the ballast decreases the period of oscillation. When we compare the maximum values, we find that if we take into account the mass of the ballast and the train, we do not find significant differences, but that is not valid at every speed. We can determine that the top speed of this train is low enough to cause few dynamic effects.
train and then ignoring it. The role of the structural damping is also addressed (0%; γ=0; 1.5%; γ=0.015). As can be seen, the structural damping does not change the dynamic effect significantly. The presence of the weight of the vehicle decreases the critical velocities where the displacements reach local maximums as does the weight of the ballast bed.

If we determine the maximum value in the case of a very low speed, we get the maximum value of the static influence line. We can choose the maximum value from the influence line belonging to any velocity. By dividing these two maximum values, the dynamic factor can be defined.

By calculating the moving mass points without damping in Fig. 14, the dynamic factor will be (at a speed of 80 m/s): \( \mu = 0.0337/0.0241 = 1.398 \).

### 6.1.2 Engine with wagons

If we can make a dynamic evaluation in the case of a single engine, there is nothing to stop us from doing the same analysis with a locomotive (M62) hauling 10 wagons. The geometry of the selected wagon can be seen in Fig. 15.

If we apply the standard to this bridge, the dynamic factor will be \( \mu = 1.17 \), which does not depend on the velocities but only on the span of the bridge. In Fig. 16, the resonance peaks appear at certain velocities.

At the actual speed range, we do not exceed this limit, but at larger speeds, especially taking into account the weight of the train and the ballast bed, we get larger factors.

![Fig. 13](image1.png) Maximum vertical displacement of the quarter point as a function of the velocity without the ballast bed

![Fig. 14](image2.png) Maximum vertical displacement of the quarter point as a function of the velocity with the ballast bed

![Fig. 15](image3.png) Tulips Hungarian wagon

![Fig. 16](image4.png) Maximum vertical displacement of the quarter point as a function of velocity
6.2 High-speed train

We carried out an analysis when the loading train is an actual high-speed vehicle, the Japanese Shinkansen express; however, this train is fictive on this railway line. The top speed of this vehicle is above 300 km/h, so visualising the maximal displacement - velocity curve up to 100 m/s is not unrealistic. In this case two points were monitored - the quarter and the middle points, as can be seen in Figs. 17 and 18.

The maximum values and the dynamic effect are different. In the case of the middle point, the dynamic factor can be calculated: \( \mu = \frac{0.0132}{0.0216} = 1.636 \) and, for the quarter point, \( \mu = \frac{0.0172}{0.0436} = 2.535 \). We can see that the static displacement is larger at the quarter point, and the dynamic factor is significantly larger.

If we want to determine the dynamic factors, we have to understand the dynamic behaviour of the structure. In the case of a simply supported bridge, the first natural frequency and eigenvector are significant in the evaluation, so it is enough to analyse the middle point. A arc bridge is more complex; therefore, it is necessary to monitor not just one point.

If we visualise the displacements of the quarter point of the main girder as a function of the first wheel position at a speed of 64 m/s (at this speed we can find the maximum value of displacement in the Fig. 18.), we can determine what happens in the case of resonance (Fig. 19).

When computing with mass points, the resonance phenomena does not occur, but at a speed of 56 m/s, we find resonance with the moving mass points, but not with the moving forces. This fact shows the importance of the model selection.

6.3 Comparing the results on different bridge models

If we compare the results calculated with the three models, no significant differences can be detected in the case of a ballast bed and a speed of 26 m/s (Fig. 20.). Consequently if we consider the displacements, it is useless to apply a very large model. A three-dimensional beam model can be suitable for calculating any elements of a structure, and the computation time is favourable, so it seems to be the optimal model. The shell model is useful for controlling the results calculated on simpler models. Furthermore, we can determine the stresses on any point of the shell. The computational time is approximately two hours at one speed. It is obvious now that the resonance curves cannot be produced in a reasonable time. In Figs. 21 and 22, we can see the results calculated with the three different vehicle models. As can be seen on Fig. 21, if we consider the vehicle as a moving dynamic system, the displacements will be less than using the simpler models.
If we calculate the vertical displacements of the quarter point of the main girder as a function of the train’s velocity and the spring stiffnesses, we get surfaces (Fig. 23). Now we can determine that the velocity and the dynamic characteristics of the train have a significant role in the response of the bridge.

For comparing the second (moving forces and mass point) and the third (moving dynamic system) vehicle models, we increased the spring stiffness over the wheels so as to be able to understand the meaning of the springs. In Fig. 24, we show the vertical displacement over different spring stiffnesses at a certain velocity. The straight line is the value calculated with the moving forces and mass points. As can be seen in Fig. 24, there is a spring stiffness value, where the two different models give us the same result. This occurs at an extremely large degree of stiffness. This means that the spring behaves like a rigid body so we get the second vehicle model where we calculated with moving forces and mass points. If the spring stiffness is very large, we have numerical problems; the displacements reach extremely large values.
6.4 Visualising the results

Using the three-dimensional shell model, we can visualise the displacements of the structure at several positions of the M62 train (Fig. 25). This can be useful for understanding what happens if the train passes over a bridge. As we can see, the quarter region of the girder moves down first when the train goes on the bridge (the deformation is asymmetrical). When it approaches the middle region, the deformation turns to symmetrical. When the train leaves the bridge, the quarter region is lifted.

Fig. 25 Displacements and longitudinal stresses of the bridge at three train positions

7 CONCLUSIONS

In this paper a train and an arc bridge interaction has been studied. We analysed the effect of moving a force group, moving a mass point and a moving train. During the calculations we used a quasi-modal analysis to calculate the eigenvectors of the arc bridge. The effects of the ballast bed were studied too. A ballast bed decreases the natural frequencies of the bridge, which can cause less fatigue and traffic noise. A ballast bed can decrease the maximum displacement values, but it strongly depends on the velocity. The weight of the ballast decreases the velocities belonging to these peaks and increases the amplitudes of the peaks. A single locomotive and a locomotive with trucks were applied as dynamic loads. Significant dynamic effects were not detected at a realistic speed range. Secondly, we tried the computation using a high-speed train. In this case we found really large effects. We constructed three bridge models. The results do not show any great differences in the results, which can be useful when applying a three-dimensional beam model, because it contains each structural element, but the computational time is acceptable. In the case of the train model, the dynamic effect was less than in the case of moving mass points, but as the stiffness of the train springs increase, the dynamic effect increases too.

REFERENCES

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