INTRODUCTION

During the structural design process, an engineer has to consider problems of the safety, reliability and durability of a single structural element as well as the entire structure from the point of view of its planned life cycle. The reliability of structural elements and systems is analyzed using two limit states – the ultimate limit state and the serviceability limit state. The loading effects are compared here with the effects of the resistance of the structure with respect to the environmental effects.

Randomness in the loading and the environmental effects, the variability of the material and geometric characteristics of structures and many other “uncertainties” affecting errors in the computing model lead to a situation where the actual behavior of a structure is different from the modeled one. Recent advances and the general accessibility of information technologies and computing techniques give rise to assumptions concerning the wider use of the probabilistic assessment of the reliability of structures through the use of simulation methods (Marek, P., Brozzetti, J., Guštar, M., 2001; Janas, P., Krejsa, M., 2002; Soares, R. C., et al., 2002; Teplý, B., Novák, D., 2004; Králik, J., Králik, J., 2006; Králik, J., Kosa, T., 2006; Králik, J., Varga, T., 2006). Much attention should be paid to using the probabilistic approach in an analysis of the reliability of structures (Ellingwood, B., et al., 1980; Rosovsky, D. V., 1995; Sadovský, Z., 2002).

Most problems concerning the reliability of building structures are defined today as a comparison of two stochastic values, loading effects \( E \) and the resistance \( R \), depending on the variable material and geometric characteristics of the structural element. The variability of those parameters is characterized by the corresponding functions of the probability density \( f_R(r) \) and \( f_E(e) \). In the case of a deterministic approach to a design, the deterministic (nominal) attributes of those parameters \( R_d \) and \( E_d \) are compared.
The deterministic definition of the reliability condition has the form

$$R_d \geq E_d$$ (1)

and in the case of the probabilistic approach, it has the form

$$RF = R - E \geq 0$$ (2)

where $RF$ is the reliability function, which can generally be expressed as a function of the stochastic parameters $X_1, X_2$ to $X_n$, used in the calculation of $R$ and $E$.

$$RF = g(X_1, X_2, ..., X_n)$$ (3)

The failure function $g(X)$ represents the condition (reserve) of the reliability, which can either be an explicit or implicit function of the stochastic parameters and can be single (defined on one cross-section) or complex (defined on several cross-sections, e.g., on a complex finite element model).

The most general form of the probabilistic reliability condition is given as follows:

$$p_f = P(R - E < 0) \equiv P(RF < 0) < p_d$$ (4)

where $p_d$ is the so-called design ("allowed" or "acceptable") value of the probability of failure. From the analytic formulation of the probability density by the functions $f_R(r)$ and $f_E(e)$ and the corresponding distribution functions $\Phi_R(x)$ and $\Phi_E(x)$, the probability of failure can be defined in the general form:

$$p_f = \int \int f_R(x) \Phi_E(x) dx = \int \int f_R(x) \Phi_E(x) dx$$ (5)

This integral can be solved analytically only for simple cases; in a general case it should be solved using numerical integration methods after discretization.

Together with $p_f$, the index of reliability $\beta$ is often used for analyzing the degree of reliability. It is defined using the linearization condition of the failure function $g(X)$. In the case of the normal distribution of this function it is valid:

$$\beta = \frac{\mu_{RF}}{\sigma_{RF}}$$ (6)

where $\mu_{RF}$ and $\sigma_{RF}$ are the mean value and the standard deviation of the reliability function, respectively.

**RELIABILITY ANALYSIS METHODS**

From the point of view of one’s approach to the values considered, structural reliability analyses can be classified in two categories, i.e., deterministic analyses and stochastic analyses. In the case of the stochastic approach, various forms of analyses (statistical analysis, sensitivity analysis, probabilistic analysis) can be performed. Considering the probabilistic procedures, Eurocode 1 recommends a 3-level reliability analysis. The reliability assessment criteria according to the reliability index are defined here. Most of these methods are based on the integration of Monte Carlo (MC) simulations. Three categories of methods have been presently realized:

- **Direct methods** (Importance Sampling - IS, Adaptive Sampling - AS, Direct Sampling - DS)
- **Modified methods** (Conditional, Latin Hypercube Sampling - LHS)
- **Approximation methods** (Response Surface Method - RSM)

A) **Direct Monte Carlo methods** are based on a simulation of the input stochastic parameters according to the expected probability distribution. The accuracy of this method depends upon the number of simulations and is expressed by the variational index:

$$\nu_{\mu} = \frac{1}{\sqrt{N_{\mu}}}$$ (7)

where $N$ is the number of simulations. If the required probability of failure is $p_f = 10^{-4}$, then by the number of simulations $N=10^6$, the variational index is equal to 10%, which is an acceptable degree of accuracy.

**Advantages of the method:**

- the final values of the reliability reserves can be continuously displayed in the form of a histogram or cumulative function; the simulations are independent,
- the method is easily understandable and transparent,
- the method enables the estimation of the statistical discrepancy of the estimation on a particular relevance level.

**Disadvantages of the method:**

- large number of simulations for small probability values,
- slow calculations of complex problems (for Finite Element Method models, the calculations are expensive and ineffective).

B) **The Modified LHS method** is based on the same number of simulations of the function $g(X)$ as in the Monte Carlo method; however the zone of the distributive function $\Phi(X)$ is divided into $N$ intervals with identical degrees of probability. This method provides good assessments of the statistical parameters of the structural...
response when compared to the Monte Carlo method. Using the LHS strategy, we get values like the reliability reserve parameter – the mean value $\bar{Z}$, the standard deviation $\sigma_z$, the slant index $\alpha_z$, the sharpness index $\varepsilon_z$, or the empirical cumulative distribution function.

The reduction of the number of simulations (tens to hundreds of simulations) means a valuable benefit from this method compared to the direct Monte Carlo method (thousands to millions of simulations).

C) Approximation methods - Response Surface Methods are based on the assumption that it is possible to define the dependency between the variable input and the output data through the approximation functions in the following form:

$$\bar{Y} = c_0 + \sum_{j=1}^{nY} c_j X_j + \sum_{j=1}^{nY} \sum_{l=1}^{nY} c_{jl} X_j X_l$$

where $c_0$ is the index of the constant member; $c_j$ are the indices of the linear member and $c_{jl}$ the indices of the quadratic member, which are given for predetermined schemes for the optimal distribution of the variables (Montgomery, Myers) or for using regression analysis after calculating the response (Neter).

Approximate polynomial coefficients are given from the condition of the error minimum, usually by the “Central Composite Design Sampling” (CCD) method or the “Box-Behnken Matrix Sampling” (BBM) method.

Advantages of the method:
- considerably less number of simulations than with the direct Monte Carlo method,
- it is possible to define dependencies using the “design experiments” method or regression analysis from the defined points in the case of improper approximation functions,
- particular simulations are independent from each other – parallel calculations can be used here.

Disadvantages of the method:
- The number of simulations depends on the number of variable input parameters; in the case of a large number of input parameters, the method is ineffective,
- the method is unsuitable in the case of discontinuous changes in the dependencies between the input and output values (e.g., the method is not suitable for resolving the stability of ideal elasto-plastic materials beyond the failure limit...).

The ANSYS Program belongs among the complex programs for solving potential problems. It contains a postprocessor, which enables the execution of the probabilistic analysis of structures. In Fig. 2, the procedural diagram sequence is presented from the structure of the model through the calculations, up to an evaluation of the probability of structural failure.

The ANSYS postprocessor enables the modeling of a structure as a solid body having a general shape (solid modeling), using Boolean operations, general spline planes (non-uniform rational B-splines), automated meshing and adaptive meshing.
The postprocessor enables the displaying of numerical results and, using APDL language (ANSYS Parametric Design Language), the compiling of the numerical results obtained. The postprocessor for the probabilistic design of structures enables the definition of random variables using standard distribution functions (normal, lognormal, exponential, beta, gamma, weibull, etc.), or externally (user-defined sampling) using other statistical programs like AntHILL or FREET. The probabilistic calculation procedures are based on Monte Carlo simulations (DS, LHS, user-defined sampling) and “Response Surface Analysis Methods (RSM)” (CCD, BBM, user-defined sampling). The statistical postprocessor compiles the results numerically and graphically in the form of histograms and cumulative distributional functions.

The sensibility postprocessor processes the data numerically and graphically and provides information about the sensitivity of the variables and about the correlation matrices.

### CBC HIGH-RISE BUILDING IN BRATISLAVA

During the process of the structural design of the CBC high-rise building in Bratislava, several problems occurred concerning the complicated hydrogeological conditions and uncertainties in the geometric and physical characteristics of the subgrade layers under the given building as well as their unfavorable effect upon stress of the main structural elements, especially the base slab. The CBC office buildings Nos. 1 and 2 are situated on Karadžičova Street in Bratislava between the building of the VUB Bank headquarters and an unused civil defense shelter. The No. 1 CBC

### Table 1 Summary of the stochastic numbers depending on the number of stochastic input variables

<table>
<thead>
<tr>
<th>Number of stochastic variables</th>
<th>Central Composite Design</th>
<th>Box-Behnken Design</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of quadratic function coefficients</td>
<td>Number of factorials f</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>N/A</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>21</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>28</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>36</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>45</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>55</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>66</td>
<td>3</td>
</tr>
</tbody>
</table>
high-rise building has groundplan proportions of 40 x 22 m and height 98.3 m. The building has three underground floors with a base gap on a level of -11.1 m.

The structure rests on a monolithic base slab with a thickness of 1.9 m. The foundation pit was stabilized with a reinforced concrete bath realized by Keller pile systems, and the ground properties were improved.

The structural system of CBC building No 1 is designed as a combined system consisting of two reinforced-concrete monolithic cores and a system of columns and concrete plates. In the bottom part of the building, on its edge, reinforced-concrete walls are designed along three sides of the building.

When comparing the various geological profiles, one can see that northward from the VUB Bank building, the amount of fine-grained deposits decreases in favour of sand. Hence, two geotechnical models were designed, to respect the geological evolution as well as the dominant service load. Model No. 1 considers the relief from the VS-1 – VS-2 – VS-3 bores and model No. 2, the relief from the VS-9 – VS-10 bores (Fig.6).

In order to calculate the dimensioning forces on the structural elements from the static loading, we start from the mean values of the mechanical soil properties defined in VS1 to VS3 and VS9 to VS10 bores (Tab.3). The improvement of the subgrade by the Keller grouting system was realized under the building (Tab.4), whereby the preferable values of the physical and mechanical properties of the sub-soil were achieved.
LOADING AND LOAD COMBINATION

The loading and load combination in the case of the deterministic assessment of the ultimate limit state of the structure is considered according to STN 73 0035 (STN ENV 1991-1), part 9.4.2, as follows:

A) Permanent and temporary design situations

\[ \sum_{j \neq i} \gamma_i G_j + \gamma_P P_k + \gamma_{Q1} Q_{j1} + \sum_{i \neq j} \gamma_{Q1} Q_{ij} \quad (9) \]

B) Seismic design situation

\[ \sum_{j \neq i} \gamma_i G_j + P_k + \gamma_{Auel} A_{uel} + \sum_{i \neq j} \gamma_{Q1} Q_{ij} \quad (10) \]

where \( G_j \) is the characteristic value of the permanent loads, \( P_k \) - the characteristic value of the prestress loading, \( Q_{ij} \) - the characteristic value of the dominant variable loading, \( Q_{ij} \) - the characteristic value of the i-variable loading, \( A_{uel} \) - the design value of the seismic loading, \( \gamma_i \) - the partial coefficient for the permanent load, \( \gamma_P \) - the value coefficient for the prestress loading, \( \gamma_{Q1} \) - the partial coefficient for the varying load i, \( \gamma \) - the value coefficient (of the building structure), \( \psi \) - the combination coefficients (according to part 9.4.4, STN 730035) according to Tab.4.

If the sensitivity of the structure affects the uncertainties in the subgrade’s stiffness, by using the deterministic approach, one can consider the alternative values of stiffness \( k_{o,min} \), \( k_{o,mean} \), \( k_{o,max} \) (where \( k_{o,mean}/k_{o,min} \approx k_{o,max}/k_{o,mean} \approx 1.5 \)) and solve the effect of the loads on several models from the defined load combinations. However, in these cases it is often difficult to analyze the effects of the individual factors upon the structural strain for more sophisticated structures. Therefore, in practice, we usually start from the absolute extremes of such solutions, and no special attention is paid to an analysis of the input values of the uncertainty effects.

VARIABILITY OF THE INPUT PARAMETERS

The uncertainty of loading, the stiffness of the sub-soil and the resistance of the structure can be effectively analyzed by a sensitivity analysis in the case of a probabilistic approach. Previously, the large number of necessary simulations as well as the high demands for the time and capacity of computers represented serious handicaps for using the direct Monte Carlo method. As long as the dependencies between the monitored values can be expressed by a continuous function, it is appropriate to use the RSM method for large-scale problems.

In the probabilistic and sensitivity analyses of the CBC high-rise building, the uncertainties of the loading and resistance are expressed depending on the characteristic values of the input values and the variable parameters based on the recommendations of JCSS [3] and ASCE 7/95 [1] (Tab.5).

The variable values of the ground’s stiffness come from geological survey results at the VS1 to VS3 and VS9 to VS10 bores and the given relief under the base slab as well as from the design of the ground improvement using the KELLER system (\( k_{z,k} = 1.5 \text{MPa/m} \)). The variability of the subgrade’s stiffness on a horizontal plane is expressed through the variable values of the global rotations \( k_{xx \_var}, k_{yy \_var} \) of the building as a solid body. The uncertainties of the calculation model were taken into account by the various parameters of the model’s uncertainties \( \theta_k \) and loading effects \( \theta_E (\mu = 1, \sigma = 0.1) \) according to a Gaussian normal distribution.

**Tab. 3** Mechanical properties of the soil layers after improving the ground

<table>
<thead>
<tr>
<th>Point</th>
<th>( Z )</th>
<th>( h_i )</th>
<th>( \gamma )</th>
<th>( E_{def} )</th>
<th>( \nu )</th>
<th>Number of material</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.97</td>
<td>1.97</td>
<td>26.3</td>
<td>155,625</td>
<td>0.3</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>-3.36</td>
<td>1.39</td>
<td>25.3</td>
<td>11,314</td>
<td>0.5</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>-11.10</td>
<td>7.74</td>
<td>26.0</td>
<td>9452</td>
<td>0.5</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>-13.80</td>
<td>2.70</td>
<td>20.0</td>
<td>75,735</td>
<td>0.2</td>
<td>19</td>
</tr>
<tr>
<td>5</td>
<td>-17.30</td>
<td>3.50</td>
<td>19.0</td>
<td>33,748</td>
<td>0.4</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>-18.30</td>
<td>1.00</td>
<td>25.0</td>
<td>900,000</td>
<td>0.2</td>
<td>21</td>
</tr>
<tr>
<td>7</td>
<td>-30.60</td>
<td>12.30</td>
<td>19.0</td>
<td>10,466</td>
<td>0.4</td>
<td>17</td>
</tr>
<tr>
<td>8</td>
<td>-35.10</td>
<td>4.50</td>
<td>19.0</td>
<td>18,692</td>
<td>0.4</td>
<td>18</td>
</tr>
<tr>
<td>9</td>
<td>-37.80</td>
<td>2.70</td>
<td>20.00</td>
<td>102,383</td>
<td>0.2</td>
<td>22</td>
</tr>
</tbody>
</table>

![Fig. 6 Geotechnical investigation of the locality](image)
Tab. 4 Probabilistic model of the principal values

<table>
<thead>
<tr>
<th>Characteristic value</th>
<th>Subsoil</th>
<th>Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stiffness characteristics</td>
<td>Permanent Variables</td>
</tr>
<tr>
<td></td>
<td>direction Z</td>
<td>around X</td>
</tr>
<tr>
<td>Variable</td>
<td>k_z_var</td>
<td>k_xx_var</td>
</tr>
<tr>
<td>Type of histogram</td>
<td>Normal</td>
<td>Normal</td>
</tr>
<tr>
<td>Mean value</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Direction of deviation</td>
<td>0.200</td>
<td>0.033</td>
</tr>
<tr>
<td>Min. value</td>
<td>0.148</td>
<td>0.851</td>
</tr>
<tr>
<td>Max. value</td>
<td>1.867</td>
<td>1.163</td>
</tr>
</tbody>
</table>

Tab. 5 Comparison of the foundation slab’s strain depending on various calculation models

<table>
<thead>
<tr>
<th>Model</th>
<th>Extreme bending moment on base slab [kNm]</th>
<th>Number of cycles</th>
<th>CPU [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
<td>Mean</td>
</tr>
<tr>
<td>Deterministic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CBC2-CBC9</td>
<td>-12,316</td>
<td>-4974</td>
<td>-</td>
</tr>
<tr>
<td>Probabilistic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CBC9p</td>
<td>-16,404</td>
<td>-10,782</td>
<td>-13,563</td>
</tr>
<tr>
<td>CBC9p2</td>
<td>-15,313</td>
<td>-4419</td>
<td>-9756</td>
</tr>
<tr>
<td>CBC9L</td>
<td>-11,664</td>
<td>-8044</td>
<td>-9761</td>
</tr>
</tbody>
</table>

Fig. 7 Distribution of points of the RSM shape for the moment M_max and displacement D_max
The probabilistic calculation was realized on the three CBC9P models (five variable parameters $k_z$, $k_{xx}$, $k_{yy}$, $\theta_R$, $\theta_E$), CBC9P2 and CBC9L (eight variable parameters $k_z$, $k_{xx}$, $k_{yy}$, $G_k$, $Q_k$, $W_k$, $\theta_R$, $\theta_E$). The output parameters were resolved by $10^6$ simulations using the Monte Carlo method. The CBC9P and CBC9P2 models were resolved using the RSM method, and the CBC9L model was resolved using the LHS method for 100 simulations.

In Figs. 7 and 8, there the distribution of the points of uncertainty for the parameters $M_{\text{max}}$ (bending moment) and $D_{\text{max}}$ (max. displacement) are displayed depending on the input parameters, which were obtained using the RSM method. From these discrete points, the development of the uncertain parameters is made, based on the relation (8) using the Monte Carlo simulations. The sensitivity analysis is based on a matrix of the correlation coefficients of the input and output stochastic parameters. The probability of failure or the reliability of the structure is obtained from the probability density of the failure functions, or reliability (Fig. 9).

CONCLUSION

The results of the calculations, which are displayed in Tab.5 and Figs. 8 and 9 show that the RSM method is more conservative than the LHS method. On the other hand, the RSM method is more effective with a smaller number of variable parameters than the LHS method.

The probabilistic approach for analyses of robust structures using FEM were presented in the paper. The effectiveness of the RSM method in combination with the Monte Carlo simulations is shown on the
example of the sensitivity analysis of the CBC high-rise building in Bratislava, and on the uncertainties in the subgrade’s stiffness and loading values, using the ANSYS software system. It should be mentioned that the RSM method is preferable to the LHS method only if the number of variables is less than 8; otherwise, the benefits of the RSM method are lost. Moreover users should be warned of the limitations in using the method (mentioned in the paper), i.e., that it is only suitable for solving problems with continuous bonds among the variable parameters (not suitable for stability problems...).

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REFERENCES

REFERENCES


