1 INTRODUCTION

The procedure we present is applicable to linear systems where the behavior of the structural elements and the soil can be represented through dynamic stiffnesses (impedances) based on the Fourier transformation of the corresponding differential equation. The explicit explanation we restrict here to axially loaded structural members and to massless rigid surface foundations. The extension to general frame structures constructed on flexible foundations embedded into a sub-grade excited by a known seismic field is straightforward if the impedance of the soil-structure interface is known from literature or is calculated by numerical procedures.

Here we consider a tower-like structure under vertical seismic excitation. Firstly, we show the effect of different material models of the structure. In this case the structure is founded on rigid bedrock to exclude the effect of interaction with the soil. Secondly, the structure is founded on sandy soil and excited through free-field motion of the site. The resulting frequency response of the top and base of the structure and the axial deformation of the structure as a function of frequencies is calculated.

2 SOIL-STRUCTURE SYSTEM

We consider a soil region which experiences seismic excitation, represented as vertically propagating pressure waves resulting in a free field denoted as $u'$. The structure founded on the soil is here assumed to be a tower-like structure represented through a vertical beam with a length $L$, longitudinal stiffness $EA$ and density $\rho$. For simplification we assume that the beam is based on a rigid massless foundation, where the effect of the embedment can be ignored. The corresponding mechanical system is shown in Fig. 1. For vertical motion the displacement of the uniform beam can be represented...
through its two longitudinal end displacements $u_s$ and $u_l$ and its corresponding stiffness $K^x$, where

$$
K^x = \begin{bmatrix}
K^x_{ss} & K^x_{sl} \\
K^x_{ls} & K^x_{ll}
\end{bmatrix}
$$

(1)

The stiffness of the massless rigid foundation in a vertical direction is denoted as $K^F$. The direct stiffness method results in the system equation (Koleková, Y. - Schmid, G., 2004)

$$
\begin{bmatrix}
K^x_{ss} & K^x_{sl} \\
K^x_{ls} & K^x_{ll}
\end{bmatrix}
\begin{bmatrix}
u_s \\
u_l
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\text{with } \rho^x = K^x_{sl} u_l
$$

(2a)

If the structure is founded on rigid rock, the displacement $u_l$ is equal to the given free field displacement, and the unknown $u_s$ can be determined from the first equation of equation (2a), which is written as

$$
K^x_{ss} u_s = \rho^x
\text{with } \rho^x = K^x_{sl} u_l
$$

(2b)

If equation (2) is understood as the Fourier transformation of the equation of motion of the system, then $K^x$ and $K^F$ are the dynamic stiffnesses (impedances) of the structure and the foundation, respectively, and $P'$ is the effective earthquake force acting on the system. These terms are, in general, complex and depend on the frequency $\omega$.

2 THE DYNAMIC BEAM ELEMENT; AXIAL DEFORMATION

Consider the beam element in Fig. 2 with a constant mass distribution $m = \rho A$, length $L$ and axial stiffness $EA$. The longitudinal forces $P_{x1}$ and $P_{x2}$ act on the end sections 1 and 2 of the element. The corresponding displacements are $u_{x1}$ and $u_{x2}$ respectively. From the dynamic equilibrium we obtain the partial differential equation of the beam vibrating in a longitudinal direction $x$

$$
\bar{m} \ddot{u}_x = EA \dot{u}_x^* \quad (3)
$$

The dots represent the derivative with respect to time and primes the derivative with respect to the $x$ coordinate. Applying the Fourier transformation to the Equation (3) and using the definition

$$
f^x = \frac{\bar{m} \bar{u}_x}{EA} = -\frac{\bar{m} \bar{\omega}^3}{EA}
$$

we obtain the ordinary differential equation

$$
f^x \ddot{\bar{u}}_x = \bar{u}_x^* \quad (4)
$$

where $^\sim$ indicates the Fourier transformed function. The general solution of equation (4) is

$$
\bar{u}_x = A_x e^{i\omega x} + B_x e^{-i\omega x} \quad (5)
$$

The constants $A_x$ and $B_x$ in equation (5) can be determined using the boundary conditions:

$$
\bar{u}_{x(0,0)} = \bar{u}_{x1} \text{ and } \bar{u}_{x(1,0)} = \bar{u}_{x2} \quad (6)
$$

Substituting these constants into Equation (5), the displacement along the beam can be expressed through its end displacements:

$$
\bar{u}_x = \frac{\bar{u}_{x1} e^{i\omega x} - \bar{u}_{x2} e^{-i\omega x}}{e^{i\omega L} - e^{-i\omega L}} \quad (7)
$$

or in matrix form as

$$
\bar{u}_x(x) = \begin{bmatrix}
e^{i\omega x} - e^{i(\omega L)} \\
e^{-i\omega x} - e^{-i(\omega L)}
\end{bmatrix} \begin{bmatrix}
\bar{u}_{x1} \\
\bar{u}_{x2}
\end{bmatrix} \quad (8)
$$
Differentiation gives the longitudinal strain along the element
\[ \varepsilon_l(x) = \frac{e^{\xi l(x)} + e^{\xi l(-x)}}{e^{\xi l} - e^{\xi l(-x)}} \left[ \frac{\tilde{u}_{l,1}}{\tilde{u}_{l,2}} \right] \]  
(9)
The normal forces at the end cross-sections follow as
\[ \tilde{p}_{x(\pm,0)} = -EA\tilde{u}_{x(\pm,0)} \quad \tilde{p}_{\gamma(\pm,0)} = EA\tilde{u}_{\gamma(\pm,0)} \]  
(10)
Through the definition of the dynamic stiffness matrix as the relation between nodal displacements and nodal forces
\[ \tilde{P}_x \tilde{u}_x = \tilde{P}_z \]  
(11)
one obtains the elements of the dynamic beam’s stiffness in the longitudinal direction
\[ \tilde{K}_{x,1} = \frac{EA(e^{\xi l} + e^{-\xi l})}{-e^{\xi l} + e^{-\xi l}} \]  
(12)
\[ \tilde{K}_{x,2} = \frac{-2EA\xi}{-e^{\xi l} + e^{-\xi l}} \]  
(13)

4 TOWER-LIKE STRUCTURE UNDER VERTICAL SEISMIC EXCITATION

4.1 FOUNDATION ON A RIGID SUB-GRADE

We assume that the tower can be modeled as a beam with constant properties along its length. As the displacement function \( \tilde{u}_l(x) \) derived in section 3 is exact for any frequency \( \omega \), it is sufficient to model the tower with one dynamic beam element. Material damping may be included as a complex E-Modulus. The real and imaginary part of the complex E-moduli for the Voigt model, ‘hysteretic’ model and Kelvin chain model used here are shown as functions of the frequency in Fig. 3.

For the numerical calculation we assume the length of the tower as \( L = 100 \text{ m} \), its density as \( \rho = 2500 \text{ kg/m}^3 \) and the elasticity modulus as \( E = 30 \text{ GPa} \). (As the weight of the structure is not considered, cross-section A does not enter into the formulation).

The eigenfrequencies of the beam based on rigid soil are:
\[ \omega_{n,5} = \frac{(2n-1)\pi}{2} \sqrt{\frac{E}{\rho L^4}} \quad n = 1, 2, \ldots \]  
(14)
The first three eigenfrequencies are (Stojanovski, K. 2006): \( \omega_{n,1} = 54.41 \text{ rd/s} \), \( \omega_{n,2} = 163.24 \text{ rd/s} \), \( \omega_{n,3} = 217.66 \text{ rd/sec} \)  
(15)

4.1.1 Material damping ignored

The frequency response of the top of the structure due to a prescribed harmonic free field motion (here, the prescribed base motion) with unit amplitude \( \tilde{u}_b \), shows the correct singularities at the eigenfrequencies (not shown here). Two displacement shapes of...
the structure for a lower (ω =30 rd/s) and a higher (ω =217.56 rd/s) frequency are shown in Fig. 4. Note the nodes in the vibration shapes for the higher frequency.

4.1.2 Material damping: Kelvin chain model

In (Hillmer, 1987), a 2-parameter material model has been developed, where the real part of the complex E-modulus increases slightly with \( \omega \). The imaginary part, starting from zero, increases rapidly to an almost constant value (see Fig. 1). The model is based on an infinite Kelvin chain. Details can be found in (Hillmer, 1987). The corresponding frequency response is given in Fig. 4. One notices the similarity with the hysteretic model. This model gives correct responses in the time domain. The displacement shapes for the higher frequency are shown in Fig 5. Here, the nodes can be approximately observed, since the damping is smaller than with the Voigt model.

5 CONCLUSION

Using dynamic elements (spectral elements) significantly reduces the number of degrees of freedom. It is shown that the Kelvin chain material damping model, which does not violate causality, produces similar results as the incorrect “hysteretic” damping model. One observes that only for undamped structures do vibration nodes exist. For higher damping, the vibration nodes disappear, and the waves start to travel along the beam.

REFERENCES